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An Infinite Horizon Army Manpower Planning model

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THESIS

AN INFINITE HORIZON ARMY MANPOWER PLANNING MODEL

by

Wade S. Yamada

June 2000

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The Army must decide on the number of officers to access, promote, and, when necessary, separate each year. This thesis develops the Infinite Horizon Manpower Planning model (IHMP), an optimization model (based on convex quadratic programming), for managing officers in the Army Competitive Category. IHMP determines the annual numbers of accessions, promotions, and separations that best meet the desired inventory targets. In addition to operational and policy constraints, IHMP incorporates the recently implemented Officer Professional Management System XXI. Because one cannot imagine a day when the Army is not needed, the thesis regards personnel management as an infinite horizon planning problem and considers several techniques to approximate infinite time. Results from IHMP help analyze two personnel issues hypothesized by Army analysts. In one case, the Army requires the number of majors in the Operations career field to be at least 95% of its target and IHMP results indicate the number of majors in other career fields are short of their targets by as much as 30%. For the other case, IHMP outputs indicate that current inventory targets are not well aligned for a 16% reduction to the overall number of officers. IHMP analyses show how to align these inventory targets for the reduced number of officers.

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AN INFINITE HORIZON ARMY MANPOWER PLANNING MODEL

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Submitted in partial fulfillment of the
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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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LIST OF ACRONYMS

1LT	First Lieutenant
2LT	Second Lieutenant
ACC	Army Competitive Category
BARON	Budget Allocation Resource of Notional Force
BG	Brigadier General
CCATS	Competitive Category Army Tracking System
COL	Colonel
CPT	Captain
DAPE-PRS	Military Strength Analysis and Forecasting Division, ODCSPER
DCSPER	Deputy Chief of Staff, Personnel
DOPMA	Defense Officer Personnel Management Act
FY	Fiscal Year
IHMP	Infinite Horizon Manpower Problem
IO	Information Operations Career Field
IS	Institutional Support Career Field
LTC	Lieutenant Colonel
MAJ	Major
ODCSPER	Office of the Deputy Chief of Staff for Personnel
OP	Operations Career Field
OPMS XXI	Officer Professional Management System XXI
OS	Operations Support Career Field
PML	Programmed or Managed Loss
TAPLIM	Total Army Personnel Life Cycle Model

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EXECUTIVE SUMMARY

To manage its inventory of 77,800 officers, the United States Army divides them into groups or competitive categories. The largest of these is the Army Competitive Category (ACC) consisting of 44,920 officers. Each year the Army must decide on the number of officers to access, promote, and, when necessary, separate in order to best meet its desired inventory targets. Generally, these targets represent the number of officers necessary for the Army to accomplish its missions. The decisions to access, promote, and separate cannot be made arbitrarily, for they must satisfy operational and congressional requirements. More recently, the Officer Professional Management System XXI imposes additional requirements on the management of ACC officers by further separating those with ranks of major and above into four career fields.

Decisions made today have consequences far into the future. For example, a second lieutenant commissioned today may still be in the Army 30 years from now. Making these decisions more complex is the fact that senior officers, e.g., lieutenant colonels and colonels, cannot be hired from outside when there is a shortage. Instead, they must be promoted through the ranks over many years.

This thesis develops an optimization model (based on convex quadratic programming), the Infinite Horizon Manpower Planning (IHMP) model, that determines the number to annually access, promote, and separate in order to minimize the differences between the officer inventory and its targets. Because it is difficult to imagine a day when the Army will not be needed, IHMP uses an infinite planning horizon. To make the model effective and efficient as a decision aid, IHMP approximates the infinite horizon

and aggregates officers according to their ranks and career fields. In the literature, there are three well-known techniques for obtaining approximate solutions to optimization problems with infinite planning horizons. They include *truncation, primal equilibrium*, and *dual equilibrium*. To this list, this thesis adds a new technique called *sampling*. Our numerical investigation demonstrates that, although more difficult to implement and modify, dual equilibrium is more robust and efficient than truncation and primal equilibrium. Dual equilibrium provides similar, if not better, solutions using essentially the same number of variables and constraints. When applicable, primal equilibrium is more efficient with sampling than without.

To illustrate the model's effectiveness, this thesis analyzes two personnel issues hypothesized by Army analysts. One deals with the transformation of the Army's brigade structure and the other deals with a possible reduction in force. For the first issue, IHMP is useful in quantifying the effects of constraining the number of majors in the Operations career field, or MAJ-OP, to be above a certain percentage of its annual inventory target. In particular, when the number of MAJ-OP must be no less than 95% of its targets, the shortages of the number of majors in the other career fields can be as much as 30%. For the other issue, IHMP outputs indicate that current inventory targets are not well aligned for a 16% reduction to the overall number of officers. Increasing the targets for captains by approximately 20% and reducing the targets for major and lieutenant colonel by a similar percentage yields better aligned targets.

I. INTRODUCTION

"Quality people are the cornerstone of today's Army, and will remain so in the future." [U.S. Army, 2000] Without soldiers to man weapon systems or to provide leadership, the Army cannot accomplish its mission to fulfill national military strategy. To support this cornerstone, the Army annually allocates the largest portion of its budget, approximately 39.6 percent in fiscal year 2001, to its Military Personnel Account (See Figure 1.1).

	Dollars	Percent
Military Personnel Account	$ 27.7	39.6%
Operation and Maintenance	$ 23.6	33.7%
Procurement	$ 9.3	13.3%
Research, Development, Test and Evaluation	$ 5.2	7.4%
Military Construction	$ 1.4	2.0%
Army Family Housing	$ 1.2	1.7%
Base Realignment and Closure	$ 0.1	0.1%
Chemical Demilitarization	$ 1.0	1.4%
Environmental Restoration	$ 0.4	0.6%
Defense Working Capital Funds, Army	$ 0.1	0.1%

Figure 1.1 Army Budget (in billion dollars) for Fiscal Year 2001. Annually, the Army allocates the largest portion of its budget to military personnel. For fiscal year 2001, the Army will allocate 39.6% of its budget to personnel.

Because the Army's Military Personnel Account is relatively large, the forecasting and monitoring of its personnel is an important aspect of budget planning and execution. The personnel proponent within the Army is the Office of the Deputy Chief of Staff for Personnel (ODCSPER). Different organizations within ODCSPER contribute to personnel management. One of these organizations, the Military Strength Analysis and Forecasting Division (DAPE-PRS), monitors and forecasts the personnel inventory for ODCSPER. DAPE-PRS currently uses decision aids implemented in a spreadsheet and

provide forecasts based on a Markov transition matrix. Although useful, these decision aids lack components that optimize management controls such as accessions and promotions to achieve desired objectives.

A. PROBLEM STATEMENT

This thesis develops an optimization model called the Infinite Horizon Manpower Planning model (IHMP) for the management of officers in the Army Competitive Category (ACC). This model determines the number of annual accessions, promotions, and, when necessary, separations to best meet the desired inventory targets. In addition to operational and congressional requirements, IHMP also incorporates the recently implemented Officer Professional Management System XXI (OPMS XXI). While suitable alternatives exist, the thesis uses a convex quadratic function to measure deviation from the inventory targets when implementing IHMP.

Because it is difficult to imagine a day when the Army will not be needed, IHMP uses an infinite planning horizon. For models similar to IHMP, a common practice is to truncate the horizon after a finite number of years. Many military manpower models reported in the literature often use a 30 year planning horizon to reduce errors from having too short a planning horizon (Grinold [1983] refers to these errors as 'end effects'). In order to reduce solution time, some models ignore end effects and use a planning horizon as short as three years. This thesis reduces end effects and the solution time for IHMP by considering several techniques for approximating the infinite horizon.

B. THESIS OUTLINE

Chapter II describes how the Army manages its officer inventory. Chapter III formulates IHMP as a convex optimization problem. Chapter IV discusses four approximation schemes, and Chapter V uses IHMP to analyze consequences of two personnel policies suggested by DAPE-PRS analysts. Finally, Chapter VI provides conclusions and recommendations for future research.

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II. OFFICER MANAGEMENT

To manage its officers, the Army separates them into groups called "competitive categories." The largest of these categories, the Army Competitive Category (ACC), consists of officers in all 16 basic branches of the Army (e.g. infantry, armor, field artillery) in the grade of second lieutenant (2LT) to colonel (COL). The remaining categories include officers in special branches such as the Chaplain Corps, Judge Advocate General, and branches of the Medical Department. Besides categories based on branch specialty, there are two other groups of officers. One group includes warrant officers and the other consists of general officers, i.e., those with a rank of brigadier general and higher.

This chapter describes the management of the ACC officer inventory under OPMS XXI, implemented in October 1998 [U.S. Army, 1998]. The first section discusses career progression for ACC officers. The next two sections describe officer management controls and goals. Section D discusses tools and decision aids currently used by or previously developed for personnel analysts in DAPE-PRS. Finally, Section E reviews related work from the literature.

A. CAREER PROGRESSION

The Army commissions new ACC officers from one of three commissioning sources: the United States Military Academy, the Reserve Officers' Training Corps, and Officer Candidate School. Newly commissioned officers have the rank of 2LT.

From year to year, officers stay in the same rank, get promoted to the next higher rank, or leave the service. As officers progress through the Army, they potentially receive promotions at certain intervals based on years of commissioned service. A 2LT receives their first promotion to first lieutenant (1LT) after 1.5 years of commissioned service. Promotion to captain (CPT) normally occurs after 3.5 years of service. After approximately ten years of service, an officer may receive promotion to major (MAJ). Under the new OPMS XXI, the Army places new majors in one of four career fields, Operations (OP), Operations Support (OS), Information Operations (IO), and Institutional Support (IS). Once assigned to a career field, officers typically remain there until retirement or separation. Within each field, officers continue to receive promotion to higher ranks at approximately the same time. Typically, promotion to lieutenant colonel (LTC) and COL occur at 16 and 21 years of service, respectively. Promotion to brigadier general must be approved by Congress.

In addition to the competitive categories, the Army also classifies its ACC officers into two broad categories. One is a group of company grade officers, i.e., those with ranks 2LT to CPT. The other consists of field grade officers or those with ranks MAJ to COL.

B. MANAGEMENT CONTROLS

The Army closely controls and monitors its officer inventory to ensure that it is properly funded and sufficient to meet its requirements. Below are three primary methods for controlling its officer inventory.

1. Accessions

As a management control, accessions provide the only source of incoming personnel for the officer inventory and to replace officers who retire or otherwise leave the service. As mentioned previously, all ACC accessions are at the rank of 2LT. In the near term, accessions certainly affect the number of lieutenants (LT) and CPT in the inventory. Because new field grade officers must be promoted from within, the number of accessions also affects the field grade inventory far in the future.

2. Promotions

Because the Army only commissions new ACC officers as 2LT, it can manage the officer inventory at higher ranks only through promotion. Promotion to 1LT is decentralized and nearly automatic. With the exception of rare instances, the Army promotes every 2LT to 1LT.

Promotion boards determine promotion to CPT and higher ranks. There is a promotion board for each rank. Each board consists of a general officer as its president and senior officers, i.e. those with one to two grades higher than the one being considered and with a minimum rank of lieutenant colonel. The promotion boards meet annually in the spring to review the records of eligible officers for promotion and select deserving officers to promote in accordance with guidance from the Chief of Staff, Army. Because

1LT and CPT are not separated into career fields, the boards for promotion to CPT and MAJ consider all eligible officers as a single group. On the other hand, the boards for promotion to LTC and COL group eligible officers by their career fields and promote officers in each field separately.

Promotion boards typically consider officers from three zones of consideration: primary, below, and above zones. To illustrate these zones, consider the officers commissioned in FY 2000 or in year group 2000. Based on a typical Army career path, these officers would be considered for promotion to CPT, MAJ, LTC, and COL in FY 2003, 2010, 2016, and 2021, respectively. During these years, officers in year group 2000 are in the 'primary zone' for promotion to the respective grades. To promote those with exemplary service records one year early, the boards also consider year group 2000 for promotion 'below the primary zone' to MAJ, LTC, and COL, in FY 2009, 2015, and 2020, respectively. (There is no below zone promotion to CPT.) For those officers not selected for promotion in the primary zone, the next board reconsiders them in the following year for 'above zone' promotion. Typically, officers passed over for promotion above the zone must leave the Army.

The boards for MAJ to COL review records and select officers for promotion from all three zones. The board for CPT does not consider promotion below the primary zone. In their selection, the boards must comply with the minimum promotion opportunity rates (or percentages) established by the 1980 Defense Officer Personnel Management Act (DOPMA) [U.S. Congress, 1981]. The promotion opportunity rate is equal to the number of officers selected for promotion from the three zones divided by the number of officers eligible in the primary zone. To ensure DOPMA compliance, the

boards use slightly higher promotion opportunity rates (See Table 2.1). For CPT, the board promotes all eligible officers who are "fully qualified", those in good standing and with acceptable evaluation reports.

	CPT	MAJ	LTC	COL
DOPMA floor	95%	80%	70%	50%
Army goal	fully qualified	85%	75%	55%

Table 2.1 DOPMA and Army Promotion rates. This table compares DOPMA and Army promotion opportunity rates, the number of promotions divided by the number eligible in the primary zone. For CPT, the Army promotes all eligible officers who are "fully qualified", those officers in good standing and with acceptable evaluation reports.

DOPMA also regulates the number of officers selected for promotion below the primary zone. In particular, the number of below zone promotions must be between 5.0 and 7.5 percent of the total promotions. There is no regulation for above zone promotions. Historically, the number of above zone promotions varies between 2 and 20 percent of the total.

3. Managed and Unmanaged Losses

Attrition and retirement reduce the officer inventory. Attrition is either by choice or by involuntary separation. Upon commissioning, officers incur a service obligation. For graduates of Officer Candidate School and the Military Academy, the obligation is three and five years, respectively [Grabski, 2000b]. Officers with Reserve Officers' Training Corps scholarships are obligated for four years and it is three years for those without. Those who complete Officer Candidate School must serve for three years. At the end of their obligation, officers may decide to leave the service. For those who stay, additional military and civilian education would incur additional service obligation. For

example, each day in a fully-funded masters' degree program translates into three days of additional service obligation.

Many officers retire from the Army at the completion of 20 years of service as LTC. However, the Army also offers incentive programs, when necessary, to encourage officers to leave the Army before completing 20 years of service. Some programs provide monetary incentive or compensation and some do not. Among those with monetary incentive, the Voluntary Separation Incentive and Selective Separation Board target CPT and MAJ. The Selective Early Retirement Board forces LTC and COL to retire early without any incentive. Voluntary Early Release Programs also offer no monetary incentive. Instead, such a program would release, e.g., a 1LT with three years of service from his or her service obligation and require them to join the Army National Guard. [Grabski, 2000a]

C. MANAGEMENT GOALS

The Personnel Manning Authorization Document sets a target number of officers for each grade and competitive category [U.S. Army, 1999]. These targets represent the number of officers required by the Army to complete its mission. In practice, the officer inventory does not meet these targets exactly. Typically, the number of officers in each grade and competitive category is either over or under the targets. However, meeting these targets for field grade officers is necessary to comply with congressional law. Most stringent is the number of COL; for it cannot exceed its own target. For LTC, the requirement is slightly more relaxed; the combined number of LTC and COL cannot exceed the sum of their targets. Likewise, the combined number of MAJ and LTC cannot

exceed the sum of their targets. The Army calls this process of satisfying field grade and congressional mandates for field grade officers the "target rolldown."

D. TOOLS AND DECISION AIDS

In ODCSPER, DAPE-PRS is directly responsible for forecasting and monitoring Army personnel inventory. To aid in their forecasting efforts, DAPE-PRS analysts have been relying on results from manpower models.

Currently, DAPE-PRS analysts utilize two spreadsheet models, developed in house, to estimate the officer inventory, the Budget Allocation Resource of Notional Force (BARON) and the Competitive Category Army Tracking System (CCATS) [Grabski, 1999]. BARON and CCATS are similar. However, BARON provides budget information not available in CCATS. Although these models are adequate, they do not provide management control decisions such as accessions and promotions that best meet the inventory targets. In addition, neither BARON nor CCATS incorporates the effects of OPMS XXI.

In the past, DAPE-PRS had to abandon two optimization models designed for officer management. The Officer Projection Aggregate Level [General Research Corporation International Inc., 1992] needed to be executed overnight on a mainframe computer and did not provide meaningful forecasts of officer inventory. A component of the Officer Aggregate system [General Research Corporation International Inc., 1998] is an optimization model implemented in AMPL [Fourer, Gay, and Kernighan, 1993], an algebraic modeling system. The Officer Aggregate system runs on a personal computer and typically takes several hours to generate the problem, solve it, and produce an output.

However, the main reason for its abandonment was the quality of its forecasts. They were not meaningful and exhibited management decisions unfamiliar to DAPE-PRS analysts [Sweetser, 1999].

E. RELATED WORK

There is a substantial body of published manpower planning research that can be categorized into stochastic or deterministic models. Most of the stochastic models (e.g. Grinold and Marshall, 1977) use Markov chains to forecast the personnel inventory in various categories.

For deterministic models, several authors formulate the problem as a linear program similar to IHMP. To forecast the Army's enlisted inventory, Holtz and Wroth [1980] propose a linear programming model called the Enlisted Loss Inventory Model – Computation of Manpower Programs using Linear Programming. This model forecasts monthly strengths, gains, and losses, for each enlisted grade over a seven year horizon. Gass et al. [1988] discuss an Army manpower model that considers a 20 year planning horizon and is appealing in that it can be modified to forecast enlisted, officer, or warrant officer personnel inventory. This model is also a linear program that attempts to minimize the weighted sums of absolute deviation from targets of different types. Unlike IHMP, Gass et al. further subdivided the inventory by year of service as well as by rank. Durso and Donahue [1995] propose another enlisted inventory model, the Total Army Personnel Life Cycle Model (TAPLIM), to forecast enlisted inventory over a 15 year planning horizon. Later, Walker [1995] extends TAPLIM's planning horizon to infinity and uses techniques described in the next chapter to compute approximate solutions to

the model. Like the objective function in Gass et al., TAPLIM minimizes the weighted sums of absolute deviation from different target types.

For the U.S. Navy, Bres et al. [1980] discuss a linear programming model that determines the optimal number of naval officer accessions and distribution plan over a 20 year planning horizon. The model groups officers according to specialty, commissioning program, and years of commissioned service. The objective function minimizes the weighted absolute deviation from strength targets. McGinnis [1996] proposes another model for the Navy implemented in Lotus 1-2-3 and it is not clear whether the model optimizes officer inventory or takes a Markov chain based approach. McGinnis uses promotion rates, continuation rates for promoted officers, and for those not selected for promotion, to develop transition matrices for officer inventory. Using these rates, McGinnis is able to calculate the officer inventory by rank, year group, and specialty, over a 10 year planning horizon. Rodgers [1991] presents an enlisted personnel planning model also for the Navy. This model is a linear program that minimizes a weighted sum of several objectives such as deviations from inventory and budget targets. The planning horizon for this model was only three years, but the model forecasts the monthly inventory of enlisted sailors.

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III. INFINITE HORIZON MANPOWER PLANNING PROBLEM

This chapter presents an infinite horizon manpower planning problem or IHMP. This model differs from the two optimization models, the Officer Projection Aggregate Level and Officer Aggregate system, mentioned in the preceding chapter in several aspects. In an effort to keep the model small and, therefore, obtain an optimal solution more quickly, IHMP keeps account of officer inventory less frequently and groups officers only by ranks and career fields. The latter is to incorporate the effects of OPMS XXI. Instead of arbitrarily truncating the planning horizon to a finite planning period, IHMP assumes that the length of the planning horizon is infinite. (Chapter IV discusses techniques for solving problems with an infinite planning horizon)

The sections below describe the necessary assumptions, the manpower planning problem and its formulation.

A. ASSUMPTIONS

IHMP relies on several assumptions to strike a balance between reality and tractability.

1. Accounting for Officer Inventory

In practice, the officer inventory changes daily. Every day there are officers rotating to different assignments, separating or retiring from the Army, and promoted to higher ranks. However, it is neither practical nor beneficial to keep account of the officer inventory at the end of each day. Instead, many organizations account for their personnel

inventory at a longer time interval, e.g., at the end of each month or quarter. In IHMP, the officer inventory is observed or calculated at the end of each fiscal year.

2. Officer Classification

IHMP groups officers according to their ranks and career fields. In contrast, many manpower models in the literature further subdivide the officer population by years of service, year groups, promotion status, and, perhaps, competitive categories.

As explained in Chapter II, the promotion from 2LT to 1LT is nearly 100% in practice. So, it is not critical to distinguish them in a model. IHMP classifies both 2LT and 1LT simply as LT.

3. Promotion

The officer classification scheme discussed earlier does not separate officers into year groups and the number of officers in the primary zones must be estimated from decision variables in the model. In year t, the primary zones for CPT, MAJ, LTC, and COL, generally consists of officers who are in year group or accessed in year (t-3), (t-10), (t-16), and (t-21), respectively.

Consider the promotion to MAJ. Nominally, a CPT must accumulate 7 years of service before being considered for promotion in the primary zone to MAJ. Thus, based on the officer classification scheme discussed earlier, the decision variable representing the number of captains at the end of year t contains the number of captains from seven different year groups. Therefore, during, e.g., FY 2008, the number of officers in the primary zone for promotion to MAJ is approximately 1/7 of the decision variable

representing the inventory of CPT at the end of FY 2007. The same is true for promotion to other ranks.

For promotion to CPT, there is an alternate, and perhaps more accurate, approach. Assuming that the planning horizon starts at the end of FY 2000 (or, equivalently at the beginning of FY 2001), the number of officers in the primary zone for promotion to CPT in FY 2004 can be estimated from the decision variable representing the number of officers who accessed in FY 2001.

It is also possible to use input data to improve the accuracy in determining the number of officers in the primary zone, especially during the early part of the planning horizon. At the beginning of FY 2001, it is possible to determine the number of officers in year groups 1980 to 2000 using historical data. Table 3.1 indicates how IHMP uses these year groups to determine the number of officers in each primary zone.

	2001	2002	2003	2004	2005	2006	2007
CPT	1998	1999	2000				
MAJ	1991	1992	1993	1994	1995	1996	1997
LTC	1985	1986	1987	1988	1989	1990	
COL	1980	1981	1982	1983	1984		

Table 3.1 Eligible Year Groups in Primary Zone. This tables illustrates year groups in the primary zone for promotion for each year in the planning horizon. For planning years that do not contain a year group, IHMP approximates the number of eligible officers in the primary zone from decision variables.

In practice, an officer may be selected for promotion by a board in one fiscal year and, because of his or her standing on the promotion list, the officer may be promoted in the next year. IHMP addresses this practice in an approximate manner by assuming that selection and promotion occurs in the same fiscal year. Users can adjust the promotion opportunity rate so that the number of officers promoted during each year approximates what occurs in practice.

4. Attrition

IHMP assumes that attrition occurs before any other personnel actions such as promotions and programmed losses. In practice, attrition can occur at any time. An officer may be dissatisfied with his and her Army career and decide to leave the service. Some officers leave even after having been selected for promotion.

B. PROBLEM DESCRIPTION

In IHMP, the main decision variables are the number of officers to access, to promote, and to become part of the programmed or managed losses (PML). During each year of the planning horizon, the objective is to minimize the difference between the officer inventory and its targets. However, the decision to access, promote, or 'PML' officers cannot be made arbitrarily. In addition to congressional mandates and Army policies, there is an underlying structure that governs how an officer's career progresses through the various ranks in the Army. This structure can be depicted as a network of nodes and arcs. In Figure 3.1, each node (or circle) in a given column represents officers in each classification at the end of a fiscal year. Arcs indicate possible movements of officers from one classification into another during a given year. Because of attrition, the number of officers that move or 'flow' out of a given node is generally less than the number that 'flows' in. For clarity, the effect of attrition is not shown in the figure.

In Figure 3.1, the arc (or, more descriptively, 'retention arc') from node LT at the end of year t-1 to node LT at the end of year t indicates that an officer remains as a LT from the end of year t-1 to the end of year t. On the other hand, the arc from LT to CPT is a 'promotion arc' and represents the fact that a LT at the end of year t-1 becomes a

CPT at the end of year *t*. Because the promotion to CPT to MAJ also involves a career field assignment, there are four promotion arcs instead of one, i.e., one promotion arc for each career field.

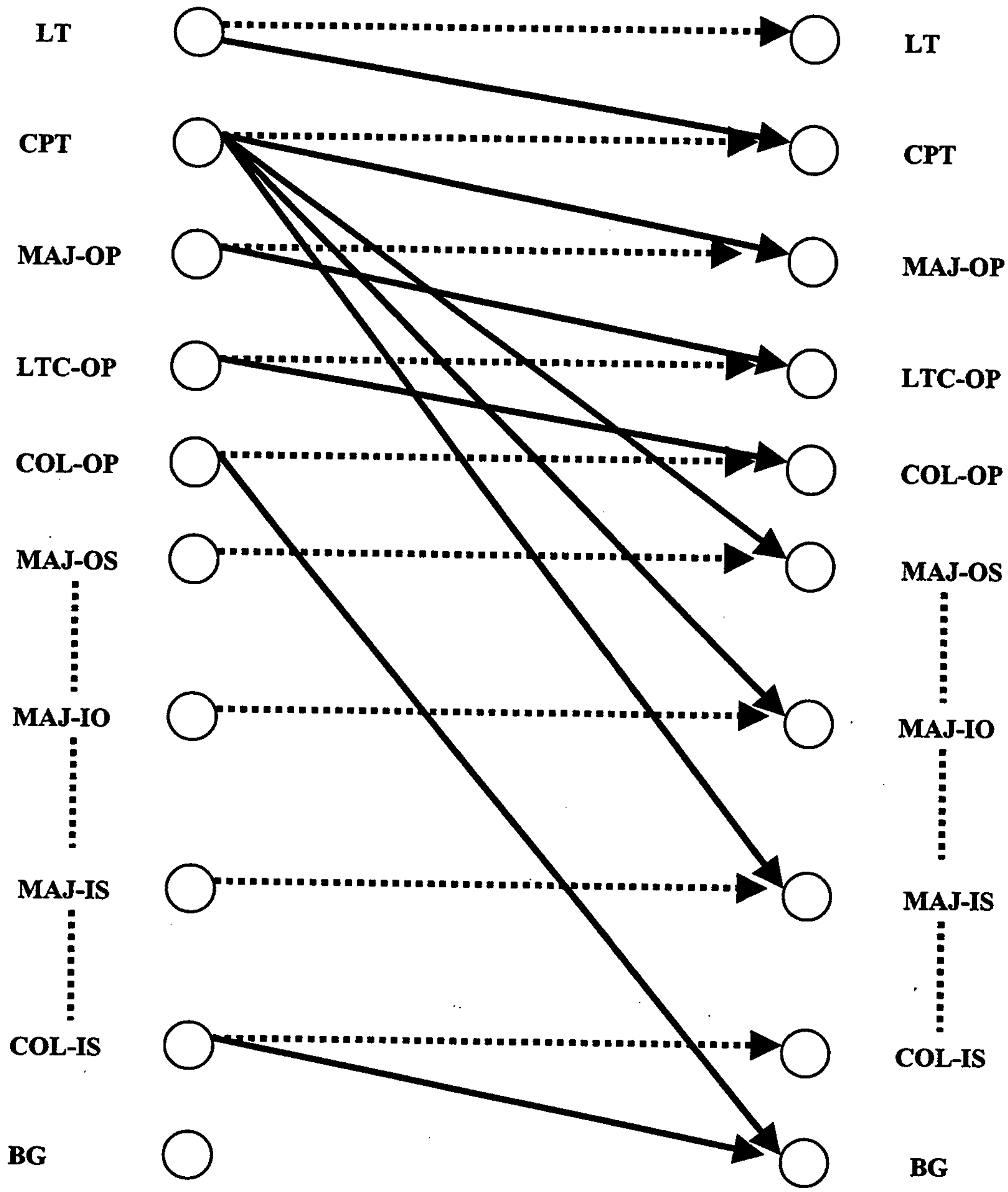

Figure 3.1 Career Progression as a Network There is an underlying network structure that governs how an officer progresses through the Army ranks. Each node (or circle) in each column represents the number of officers based on the classification scheme at the end of the year. Arcs represent the movement of officers from one year to the next. Dotted arcs or 'retention' arcs represent officers remaining in the same rank from the end of year t-1 to the end of year t. Solid arcs or 'promotion' arcs correspond to officers promoted to higher ranks. Promotion from CPT to MAJ also involves career field designation.

C. PROBLEM FORMULATION

Below is a formulation of the infinite horizon manpower planning problem.

Indices:

t	year of planning horizon, $t = 1, 2, 3, \ldots \infty$
yos	years of commissioned service
r	rank, $r = LT, CPT, MAJ, LTC, COL, BG$
c	career field, $c = NA, OP, OS, IO, IS$, where NA means "not applicable. In particular, lieutenants and captains have no career field.

The following derived set limits the range of indices on decision variables to only those that are logical.

Derived Set

Ω	valid combinations of rank r and career field c to track officer inventory, i.e., $\{(LT,NA),(CPT,NA),(MAJ,NA),(MAJ,OP),\ldots,(MAJ,IS),$ $(LTC,OP),\ldots,(LTC,IS),(COL,OP),\ldots,(COL,IS)\}$. Note that ($MAJ$, NA) refers to majors prior to their career field assignment.

Data:

α	discount factor, i.e., $0 < \alpha < 1$
$\tau_{r,c}$	the year, for $t = 1,\ldots, \tau_{r,c}$, where the number of officers for rank r, career field c, is derived from data rather than approximated from inventory variables (defined below).
$\underline{acc}_t$, $\overline{acc}_t$	minimum and maximum number of lieutenants to access in year t. For some t, $\underline{acc}_t$ and $\overline{acc}_t$ may be the same.
$\underline{azp}_r$, $\overline{azp}_r$	minimum and maximum proportion of officers who can be promoted from the above zone to rank r
bg_t	the number of brigadier generals selected in year t, from the colonel population (number of officers)

$\underline{bzp}_r$, $\overline{bzp}_r$	minimum and maximum proportion of officers who are promoted from the below zone to rank r
cf_c	minimum proportion of new majors assigned to career field c
cr_{yos}	proportion of officers who remain in the Army (or survival rate) after yos years of service.
fyg_r	proportion of officers eligible for promotion in the primary zone for rank r
$lr_t^{r,c}$	proportion of officers in rank r, career field c, who attrit during year t
$\overline{pmlp}_r$	maximum proportion of officers in rank r who are forced to separate
$\underline{pr}_r$, $\overline{pr}_r$	minimum and maximum promotion opportunity rate for promotion to rank r
$pze_t^{r,c}$	number of officers in the primary zone of promotion for rank r, career field c, for $t = 1,\ldots,\tau_{r,c}$.
$tgt_t^{r,c}$	targeted number of officers in rank r and career field c, at the end of year t

Nonnegative Variables:

$X_t^{r,c}$	number of officers at rank r, in career field c, at the end of year t. To simplify the inventory balance constraints below, $X_0^{r,c}$ is not a decision variable. Instead, it represents an initial number of officers in rank r and career field c. Henceforth, it is sometimes convenient to refer to $X_t^{r,c}$ collectively as inventory variables.
$PZ_t^{r,c}$	number of officers promoted from the primary zone to rank r and in career field c during year t
$BZ_t^{r,c}$	number of officers promoted from the below zone to rank r and in career field c during year t
$AZ_t^{r,c}$	number of officers promoted from the above zone to rank r and in career field c during year t
$PML_t^{r,c}$	number of officers in rank r and career field c, who are forced to separate from the Army during year t
A_t	number of lieutenants accessed into the Army during year t
CF_t^c	number of officers promoted to major in career field c during year t

Infinite Horizon Manpower Planning Model

Formulation:

$$\text{Min} \sum_{t=1}^{\infty} \sum_{(r,c)\in\Omega} \alpha^t f_t^{r,c}(X_t^{r,c})$$

Subject to

Inventory constraints

$$X_t^{LT,NA} - (1 - lr_t^{LT,NA}) \cdot X_{t-1}^{LT,NA} - A_t + PZ_t^{CPT,NA} + AZ_t^{CPT,NA} + PML_t^{LT,NA} = 0 \qquad \forall\, t \quad (3.1a)$$

$$X_t^{CPT,NA} - (1 - lr_t^{CPT,NA}) \cdot X_{t-1}^{CPT,NA} - PZ_t^{CPT,NA} - AZ_t^{CPT,NA} + BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA} + PML_t^{CPT,NA} = 0 \qquad \forall\, t \quad (3.1b)$$

$$X_t^{MAJ,c} - (1 - lr_t^{MAJ,c}) \cdot X_{t-1}^{MAJ,c} - CF_t^c + BZ_t^{LTC,c} + PZ_t^{LTC,c} + AZ_t^{LTC,c} + PML_t^{MAJ,c} = 0 \qquad \forall\, t,\, c \neq NA \quad (3.1c)$$

$$X_t^{LTC,c} - (1 - lr_t^{LTC,c}) \cdot X_{t-1}^{LTC,c} - BZ_t^{LTC,c} - PZ_t^{LTC,c} - AZ_t^{LTC,c} + BZ_t^{COL,c} + PZ_t^{COL,c} + AZ_t^{COL,c} + PML_t^{LTC,c} = 0 \qquad \forall\, t,\, c \neq NA \quad (3.1d)$$

$$X_t^{COL,c} - (1 - lr_t^{COL,c}) \cdot X_{t-1}^{COL,c} - BZ_t^{COL,c} - PZ_t^{COL,c} - AZ_t^{COL,c} + bg_t, + PML_t^{COL,c} = 0 \qquad \forall\, t,\, c \neq NA \quad (3.1e)$$

Promotion opportunity constraints

$$\underline{pr}_r \cdot (pze_t^{r,c} - BZ_{t-1}^{r,c}) \leq (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq \overline{pr}_r \cdot (pze_{t,yg}^{r,c} - BZ_{t-1}^{r,c})$$
$$\forall\, (r, c) \in \Omega,\, (r,c) \neq \{(LT,NA), (CPT,NA), (MAJ,OP), \ldots, (MAJ,IS)\},\, t \leq \tau_{r,c} \quad (3.2a)$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{t-3} \leq (PZ_t^{CPT,NA} + AZ_t^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{t-3} \qquad \forall\, t > \tau_{CPT,NA} \quad (3.2b)$$

$$\underline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_{t-1}^{CPT,NA} \leq (BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA}) \leq \overline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_{t-1}^{CPT,NA}$$
$$\forall\, t > \tau_{MAJ,NA} \quad (3.2c)$$

$$\underline{pr}_r \cdot fyg_r \cdot X_{t-1}^{r-1,c} \leq (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq \overline{pr}_r \cdot fyg_r \cdot X_{t-1}^{r-1,c}$$
$$\forall\, r \in \{LTC, COL\},\, c \neq NA,\, t > \tau_{r,c} \quad (3.2d)$$

Below and Above zone promotion constraints

$$\underline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \le BZ_t^{r,c} \le \overline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \ne \{(LT,NA),\ (CPT,NA),(MAJ,OP),\ldots,(MAJ,IS)\},\ t \ge 2 \quad (3.3a)$$

$$\underline{azp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \le AZ_t^{r,c} \le \overline{azp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \ne \{(LT,NA),(MAJ,OP),\ldots,(MAJ,IS)\},\ t \ge 2 \quad (3.3b)$$

Career field accession constraints

$$\sum_{c \ne NA} CF_t^c - BZ_t^{MAJ,NA} - PZ_t^{MAJ,NA} - AZ_t^{MAJ,NA} = 0 \qquad \forall\ t \ge 2 \quad (3.4a)$$

$$CF_t^c \ge cf_c \cdot (BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA}) \qquad \forall\ t \ge 2,\ c \ne NA \quad (3.4b)$$

Rolldown constraints

$$\sum_{c \ne NA} X_t^{COL,c} \le \sum_{c \ne NA} tgt_t^{COL,c} \qquad \forall\ t \ge 2 \quad (3.5a)$$

$$\sum_{c \ne NA} X_t^{LTC,c} + \sum_{c \ne NA} X_t^{COL,c} \le \sum_{c \ne NA} tgt_t^{LTC,c} + \sum_{c \ne NA} tgt_t^{COL,c} \qquad \forall\ t \ge 2 \quad (3.5b)$$

$$\sum_{c \ne NA} X_t^{MAJ,c} + \sum_{c \ne NA} X_t^{LTC,c} \le \sum_{c \ne NA} tgt_t^{MAJ,NA} + \sum_{c \ne NA} tgt_t^{LTC,c} \qquad \forall\ t \ge 2 \quad (3.5c)$$

Program managed loss constraints

$$\sum_{\substack{(r,c)\in\Omega,\\ (r,c)\ne(MAJ,NA)}} PML_t^{r,c} \le \overline{pmlp}_r \cdot \sum_{\substack{(r,c)\in\Omega\\ (r,c)\ne(MAJ,NA)}} X_t^{r,c} \qquad \forall\ t \ge 2 \quad (3.6)$$

Accession constraints

$$\underline{acc}_t \le A_t \le \overline{acc}_t \qquad \forall\ t \ge 2 \quad (3.7)$$

Nonnegativity constraints

$$X_t^{r,c},\ PZ_t^{r,c},\ BZ_t^{r,c},\ AZ_t^{r,c},\ PML_t^{r,c},\ A_t,\ CF_t^c \ge 0 \qquad \forall\ r,\ c,\ t \quad (3.8)$$

In the objective function, $f_t^{r,c}(X_t^{r,c})$ measures the deviation of $X_t^{r,c}$ from its targets and α^t is a discount factor. This thesis uses $f_t^{r,c}(X_t^{r,c}) = w_t^{r,c}(X_t^{r,c} - tgt_t^{r,c})^2$, where $w_t^{r,c} > 0$ is a weight associated with $X_t^{r,c}$. Defined in this manner, $f_t^{r,c}(X_t^{r,c})$ is quadratic and convex. Another common function for measuring deviation is $f_t^{r,c}(X_t^{r,c}) = w_t^{r,c}\left|X_t^{r,c} - tgt_t^{r,c}\right|$, where $w_t^{r,c}$ is as previously defined. In this case, the resulting problem has an equivalent linear programming formulation with additional variables and constraints.

The inventory balance constraints, equations (3.1a) to (3.1e) relates the officer inventory from one year to the next. In words, these constraints state that the number of officers in rank *r*, career field *c*, at the end of year *t*

= the number of officers in rank *r*, career field *c* who survive from the end of year *t*-1

\+ the number of officers promoted to rank *r*, career field *c*, during year *t*

\- the number of officers promoted to rank *r*+1, career field *c*, during year *t*

\- those who separate from the Army as PML.

Recall from Chapter 2 that the promotion opportunity rate for each rank equals the total number of promotions in all three promotion zones divided by the number of officers in the primary zone. Constraints in equations (3.2a) to (3.2d) ensure that these rates are within their bounds. In particular, the bounds in equation (3.2a) are for promotion in the early part of the planning horizon ($t = 1, \ldots \tau_{r,c}$) and they depend on those officers who joined the Army prior to the beginning of the horizon. For better

accuracy, these bounds are computed from data, $pze_t^{r,c}$, instead of inventory variables, $X_t^{r,c}$, in the model.

In general, the bounds on the promotion opportunity to CPT depend on the number of lieutenants with three years of service. Equation (3.2b) approximates this number in year t by multiplying the number of officers who accessed in year (t-3) with the appropriate survival rate, cr_3. For higher ranks, the bounds on the promotion opportunity are functions of the number of officers in the primary zone. In equation (3.2c) and (3.2d), this number is taken to be a fraction, fyg_r, of the appropriate inventory variable.

Similarly, equations (3.3a) and (3.3b) ensure that below and above zone promotions are within appropriate bounds. Congressional mandates determine bounds for below zone promotion rates. The bounds for above zones are for management purposes.

Equations (3.4a) and (3.4b), allocate newly promoted majors to the four career fields. Mathematically, (3.4a) ensures that the total number of accessions into each career field is equal to the total number of promotions in all three zones. On the other hand, (3.4b) guarantees that each career field accesses a minimum number of officers.

Equations (3.5a) to (3.5c) implement target rolldown (see Chapter II). Equation (3.5a) does not allow the COL inventory to exceed its target. Equation (3.5b) ensures that the number of LTC and COL does not exceed their combined targets. Likewise, equation (3.5c) is for MAJ and LTC.

Constraints in equation (3.6) prevent the number of programmed or managed losses within each rank from exceeding a maximum proportion.

Constraints in equation (3.7) force the annual number of accessions to be within the desired upper and lower bounds. In practice, it is possible that $\underline{acc}_t$ equals $\overline{acc}_t$ to reflect the fact that the numbers of graduates from all commissioning programs are known with near certainty during the early part of the planning horizon.

Finally, constraints in equation (3.8) qualify all decision variables as nonnegative.

IV. APPROXIMATION SCHEMES FOR INFINITE HORIZONS

In practice, it is not possible to solve the infinite horizon manpower planning (IHMP) problem because it has an infinite number of constraints and variables. Grinold [1983] (see also Walker, 1995) proposes several techniques or schemes for obtaining approximate solutions to optimization problems with an infinite number of variables and constraints. Three schemes in Grinold [1983] are applicable to IHMP and they include *truncation*, *primal equilibrium*, and *dual equilibrium*. When combined with these three schemes, a fourth scheme, called *sampling*, is helpful in further reducing the size of the resulting problem.

A. TRUNCATION

Truncation is the simplest approximation scheme to implement. It simply discards decision variables and constraints after a truncation year T. Implicitly, truncation assumes that the Army is in existence only up to year T. Because the truncated model is less constrained and does not account for target deviation after the truncation year, it provides a lower bound for IHMP.

The quality of the solution obtained via truncation depends on the value of T. A small truncation year produces a model with fewer constraints and variables. However, because a large part of the true model has been removed, the solution may be far from optimal. On the other hand, a large truncation year generally produces a better solution. However, the resulting model contains more variables and constraints and requires more time to solve.

B. PRIMAL EQUILIBRIUM

The primal equilibrium approximation scheme assumes the targets and optimal values for the decision variables reach their equilibrium values in year T or the equilibrium year, e.g., $x_t = x_T$, for $t \geq T$, where x_t represents a generic decision variable for year t. Under this assumption, IHMP reduces to a problem with a finite number of variables and constraints. Appendix A provides a complete formulation of the problem resulting from the primal equilibrium approximation scheme. Below are some of its key ideas.

Recall from Chapter 3 that the objective function for IHMP is to minimize the following expression

$$\sum_{t=1}^{\infty} \sum_{(r,c)\in\Omega} \alpha^t f_t^{r,c}(X_t^{r,c}).$$

However the above can be written as

$$\begin{aligned}
\sum_{t=1}^{\infty} \sum_{(r,c)\in\Omega} \alpha^t f_t^{r,c}(X_t^{r,c}) &= \sum_{(r,c)\in\Omega} \left[\sum_{t=1}^{T-1} \alpha^t f_t^{r,c}(X_t^{r,c}) + \sum_{t=T}^{\infty} \alpha^t f_t^{r,c}(X_t^{r,c}) \right] \\
&= \sum_{(r,c)\in\Omega} \left[\sum_{t=1}^{T-1} \alpha^t f_t^{r,c}(X_t^{r,c}) + \frac{\alpha^T}{1-\alpha} f_T^{r,c}(X_T^{r,c}) \right]
\end{aligned}$$

where the last equality follows from the following

a. $\sum_{t=T}^{\infty} \alpha^t = \dfrac{\alpha^T}{1-\alpha}$

b. $X_t^{r,c} = X_T^{r,c}$, for $t \geq T$

c. $f_t(x) = f_T(x)$, for $t \geq T$.

(Note: It is assumed here and elsewhere in the chapter that $f_t(x) = |x - target_t|$ or $(x - target_t)^2$ and $target_t = target_T$, for $t = T, (T + 1), \ldots\infty$.)

In IHMP, equation (3.1a) for any $t \geq T + 1$, can be explicitly written as

$$X_t^{LT,NA} = (1 - lr_t^{LT,NA}) \cdot X_{t-1}^{LT,NA} + A_t - PZ_t^{CPT,NA} - AZ_t^{CPT,NA} - PML_t^{LT,NA}$$

Because $x_t = x_T$, for all $t \geq T + 1$, the above reduces to

$$X_T^{LT,NA} = (1 - lr_T^{LT,NA}) \cdot X_T^{LT,NA} + A_T - PZ_T^{CPT,NA} - AZ_T^{CPT,NA} - PML_T^{LT,NA} \qquad (4.1)$$

Therefore, equation (4.1) replaces equation (3.1a) for $t \geq (T + 1), \ldots\infty$, in IHMP.

Equation (3.2b) can be written as follows

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{T-3} \leq (PZ_T^{CPT,NA} + AZ_T^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{T-3}, \qquad t = T$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{T-2} \leq (PZ_{T+1}^{CPT,NA} + AZ_{T+1}^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{T-2}, \qquad t = T + 1$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{T-1} \leq (PZ_{T+2}^{CPT,NA} + AZ_{T+2}^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{T-1}, \qquad t = T + 2$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_T \leq (PZ_{T+3}^{CPT,NA} + AZ_{T+3}^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_T, \qquad t \geq T + 3$$

Because $PZ_t^{CPT,NA} = PZ_T^{CPT,NA}$ and $A_t = A_T$, for $t \geq T$, the above equations reduce to

$$\underline{pr}_{CPT} \cdot cr_3 \cdot \max\{A_{T-3}, A_{T-2}, A_{T-1}, A_T\} \leq (PZ_T^{CPT,NA} + AZ_T^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot \min\{A_{T-3}, A_{T-2}, A_{T-1}, A_T\}$$

C. DUAL EQUILIBRIUM

The dual equilibrium approximation scheme reduces IHMP to a problem with a finite number of variables and constraints via aggregation past year T, or the aggregation year. As before, let x_t denote a generic decision variable in year t. Then for the dual equilibrium, $\bar{x}_T$ represents a convex combination of x_t, for $t \geq T$. More specifically,

$$\bar{x}_T = \sum_{t=T}^{\infty} (1-\alpha)\alpha^{t-T} x_t . \tag{4.2}$$

Observe that the weight for x_t, $(1-\alpha)\alpha^{t-T}$, is positive and sums to one as follows

$$\sum_{t=T}^{\infty} (1-\alpha)\alpha^{t-T} = (1-\alpha)\sum_{t=T}^{\infty} \alpha^{t-T} = (1-\alpha)\frac{1}{1-\alpha} = 1 .$$

Using the above form of variable aggregation, the objective function of IHMP can be approximated using a function with a finite number of terms as follows

$$\begin{aligned}
\sum_{t=1}^{\infty} \sum_{(r,c)\in\Omega} \alpha^t f_t^{r,c}(X_t^{r,c}) &= \sum_{(r,c)\in\Omega} \left[\sum_{t=1}^{T-1} \alpha^t f_t^{r,c}(X_t^{r,c}) + \sum_{t=T}^{\infty} \alpha^t f_t^{r,c}(X_t^{r,c}) \right] \\
&= \sum_{(r,c)\in\Omega} \left[\sum_{t=1}^{T-1} \alpha^t f_t^{r,c}(X_t^{r,c}) + \frac{\alpha^T}{1-\alpha} \sum_{t=T}^{\infty} (1-\alpha)\alpha^{t-T} f_t(X_t^{r,c}) \right] \\
&\geq \sum_{(r,c)\in\Omega} \left[\sum_{t=1}^{T-1} \alpha^t f_t^{r,c}(X_t^{r,c}) + \frac{\alpha^T}{1-\alpha} f_t\left(\sum_{t=T}^{\infty} (1-\alpha)\alpha^{t-T} X_t^{r,c} \right) \right] \\
&= \sum_{(r,c)\in\Omega} \left[\sum_{t=1}^{T-1} \alpha^t f_t^{r,c}(X_t^{r,c}) + \frac{\alpha^T}{1-\alpha} f_T(\overline{X}_T^{r,c}) \right]
\end{aligned}$$

where the inequality follows from the convexity of f_t and $\overline{X}_T^{r,c}$ is the aggregation or convex combination of $X_T^{r,c}$, for $t \geq T$, using equation (4.2).

Dual equilibrium replaces a collection of constraints for year $t \geq T$ with one constraint representing its convex combination. In the simplest case, consider the following (equation 3.3a)

$$\underline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq BZ_t^{r,c} \leq \overline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \ \forall\, t \geq T.$$

Multiplying each constraint by a weight of $(1-\alpha)\alpha^{t-T}$ and summing them together yields

$$\underline{bzp}_r \cdot (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c}) \leq \overline{BZ}_T^{r,c} \leq \overline{bzp}_r \cdot (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c})$$

where $\overline{BZ}_T^{r,c}$, $\overline{PZ}_T^{r,c}$, and $\overline{AZ}_T^{r,c}$ are defined in the manner of equation (4.2).

For a slightly more complicated case, consider equation (3.1a), which can be written as

$$0 = -(1 - lr_t^{LT,NA})X_{t-1}^{LT,NA} + X_t^{LT,NA} - A_t + PZ_t^{CPT,NA} + AZ_t^{CPT,NA} + PML_t^{LT,NA} \ \forall\, t \geq T.$$

As before, multiplying each constraint by a weight of $(1-\alpha)\alpha^{t-T}$ and summing them together yields

$$0 = -(1 - lr_T^{LT,NA})\left[(1-\alpha)X_{T-1}^{LT,NA} + \alpha\sum_{t=T}^{\infty}(1-\alpha)\alpha^{t-T}X_t^{LT,NA}\right]$$

$$+ \sum_{t=T}^{\infty}(1-\alpha)\alpha^{t-T}(-A_t + PZ_t^{CPT,NA} + AZ_t^{CPT,NA} + PML_t^{LT,NA})$$

$$0 = -(1 - lr_T^{LT,NA})\left[(1-\alpha)X_{T-1}^{LT,NA} + \alpha\overline{X}_T^{LT,NA}\right]$$

$$-\overline{A}_T + \overline{PZ}_T^{CPT,NA} + \overline{AZ}_T^{CPT,NA} + \overline{PML}_T^{LT,NA}$$

where $\overline{X}_T^{LT,NA}$, $\overline{A}_T$, $\overline{PZ}_T^{LT,NA}$, $\overline{AZ}_T^{LT,NA}$, and $\overline{PML}_T^{LT,NA}$ are defined in the manner of equation (4.2).

Finally, the aggregation of other constraints are similar and the resulting model using dual equilibrium approximation is given in Appendix B.

D. SAMPLING

When used in conjunction with the above three approximation schemes the 'sampling' scheme can further reduce the number of variables and constraints. In this scheme, a sample refers to a subset of years remaining after truncation, primal, or dual equilibrium approximation. The constraints corresponding to years not in the sample are discarded. Similarly, values of variables for years not in the sample are forced to be the same as those in the sample. For example, consider a sample consisting of years 1, 2, 3, 4, 5, 10, 15, and 20. The resulting model based on this sample would consist of decision variables and constraints for those years in the sample. The decision variables for years 6 to 9 are assumed to have the same values as those in year 5. Similarly, decision variables for years 11 to 14 and 16 to 19 are assumed to be the same as those in year 10 and 15, respectively. Depending on the original approximation scheme, the variables and constraints for year 21 and later can be truncated or appropriately aggregated.

Intuitively, the scheme should work well when the sampling frequency depends on or reflects the stability of the optimal values of the decision variables. Infrequent sampling during a period in which the optimal values fluctuate wildly would produce a poor result. In fact, the next chapter assesses the quality of these approximation schemes using data provided by DAPE-PRS.

V. RESULTS AND APPLICATIONS

The Chapter IV approximation schemes are implemented in the General Algebraic Modeling System (GAMS) [Brooke et al., 1997] and solved using Minos, Version 5.4 (see e.g., Murtagh and Saunders, 1987), on a 333 megahertz Pentium II, personal computer, with 64 megabytes of random access memory. The sections below describe data provided by DAPE-PRS, a numerical comparison of the approximation schemes, and analysis of two personnel issues posed by DAPE-PRS.

A. INPUT DATA

Below is a list of data provided by DAPE-PRS. Because the Army has not completely transitioned to OPMS XXI, this thesis constructs data for the four career fields from incomplete historical data with guidance from DAPE-PRS analysts.

1. Targets and Inventories

DAPE-PRS provides the targets for officer inventory for the next seven years. (see Table 5.1) This thesis assumes that targets for FY 2008 and beyond are the same as those in FY 2007. Because field grade officer targets are not separated into career fields, this thesis uses historical rates (or percentages) from Mattes [2000] to separate them. These rates are 75%, 12%, 6%, and 7%, for OP, OS, IO, and IS, respectively.

	Fiscal Year						
	2001	2002	2003	2004	2005	2006	≥ 2007
LT	12830	12832	12832	12833	12833	12833	12833
CPT	16835	16837	16838	16838	16838	16838	16838
MAJ-OP	7681	7681	7681	7681	7681	7681	7681
MAJ-OS	1229	1229	1229	1229	1229	1229	1229
MAJ-IO	614	614	614	614	614	614	614
MAJ-IS	717	717	717	717	717	717	717
LTC-OP	4932	4932	4932	4932	4932	4932	4932
LTC-OS	789	789	789	789	789	789	789
LTC-IO	395	395	395	395	395	395	395
LTC-IS	460	460	460	460	460	460	460
COL-OP	1730	1730	1730	1730	1730	1730	1730
COL-OS	277	277	277	277	277	277	277
COL-IO	138	138	138	138	138	138	138
COL-IS	161	161	161	161	161	161	161
TOTAL	48788	48791	48793	48793	48793	48793	48793

Table 5.1 Inventory Targets. DAPE-PRS' inventory targets for the next seven years. This thesis uses historical designation rates (percentages) to separate field grade officers into four career fields. These rates are 75%, 12%, 6%, and 9%, for OP, OS, IO, and IS, respectively.

The initial inventory for our evaluation and applications is the officer inventory at the end of FY 2000. Table 5.2 displays this inventory as projected by DAPE-PRS using the above designation rates.

	NA	OP	OS	IO	IS
LT	14145	0	0	0	0
CPT	15549	0	0	0	0
MAJ	0	7608	1217	609	710
LTC	0	4647	743	372	434
COL	0	1553	248	124	145

Table 5.2 The Officer Inventory at the End of FY 2000. The above are estimates of officers in each rank and career field based on DAPE-PRS' projection and historical designation rates.

2. Attrition Rates

Table 5.3 displays the attrition rates for FY 2001 to FY 2007 used by DAPE-PRS; attrition rates for FY 2008 and beyond are assumed to be the same as those in FY 2007. Also, because the Army has not completely transitioned to OPMS XXI, there is no attrition data available for the different career fields. Therefore, this thesis assumes that the attrition rates for field grade officers are the same for all career fields.

	2001	2002	2003	2004	2005	2006	≥ 2007
LT	4.4%	4.5%	4.5%	4.4%	4.4%	4.4%	4.4%
CPT	14.0%	13.9%	13.9%	13.9%	13.9%	13.9%	13.9%
MAJ-OP	7.6%	6.2%	5.7%	6.2%	6.2%	6.2%	6.2%
MAJ-OS	7.6%	6.2%	5.7%	6.2%	6.2%	6.2%	6.2%
MAJ-IO	7.6%	6.2%	5.7%	6.2%	6.2%	6.2%	6.2%
MAJ-IS	7.6%	6.2%	5.7%	6.2%	6.2%	6.2%	6.2%
LTC-OP	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%
LTC-OS	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%
LTC-IO	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%
LTC-IS	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%	12.9%
COL-OP	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%
COL-OS	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%
COL-IO	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%
COL-IS	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%	19.4%

Table 5.3 Attrition Rates. DAPE-PRS' forecasted attrition rates for the next seven years. For all ranks except for MAJ, attrition is fairly constant. Because career field based attrition rates are not available, all career fields for MAJ, LTC, and COL, have the same attrition rate.

3. Bounds for Promotion Rates

The lower bounds on promotion opportunity rates are from DOPMA. In Table 5.4, the upper bounds are 10% above the lower bounds and bounds for below and above zone promotions are from DAPE-PRS.

	Promotion Opportunity Rate		Below Zone Rates		Above Zone Rates	
	Lower	Upper	Lower	Upper	Lower	Upper
CPT	95%	100%	NA	NA	0.8%	2.0%
MAJ	80%	90%	6.5%	7.5%	0.8%	2.0%
LTC	60%	70%	6.5%	7.5%	3.0%	5.0%
COL	50%	60%	6.5%	7.5%	3.0%	12.0%

Table 5.4 Bounds on Promotion rates. The above table displays the upper and lower bounds used to limit the number of primary, below, and above zone promotions. The lower bounds for the promotion opportunity rates are from DOPMA, while the upper bounds are 10% above DOPMA for all ranks except for promotion to CPT. For the above and below zone rates, their bounds are either from DOPMA or based on guidance from DAPE-PRS.

4. Accessions

Following DAPE-PRS practice, officer accessions between FY 2001 and FY 2004 are set at 4,100, and, for FY 2005, it is 4,300. For FY 2006 and beyond, the lower and upper bounds for officer accessions are 3,500 and 4,500, respectively.

B. NUMERICAL COMPARISON

The four approximation schemes discussed earlier contain parameters that can control or influence the quality of the approximation. For truncation, primal and dual equilibrium, Grinold [1983] (see also Walker, 1995) demonstrated that small T values (which represents the truncation, equilibrium, and aggregation year, respectively) would result in larger end effects, i.e., errors due to having too short a planning horizon. For the sampling technique, infrequent sampling would cause similar errors. In addition, the discount factor, α, also plays a role in causing end effects. Using the data described in Section A, this section examines the trade-off between end effects and the two parameters, T and α.

Recall that the objective function of IHMP is of the form $\sum_{t=1}^{\infty} \sum_{(r,c)\in\Omega} \alpha^t f_t^{r,c}(X_t^{r,c})$,

where $f_t^{r,c}\left(X_t^{r,c}\right) = w_t^{r,c}\left(X_t^{r,c} - tgt_t^{r,c}\right)^2$, and different values of α^t would yield different weights for these deviations. Figure 5.1 graphically displays the values of α^t for three different α values, 0.5, 0.725, and 0.95, where 0.725 is the midpoint of the interval [0.5, 0.95]. When $\alpha = 0.5$, α^t decreases to zero quickly and essentially discounts the deviations from year 8 and beyond as unimportant. For $\alpha = 0.725$, this is also true, but around year 16. For $\alpha = 0.95$, the deviations for future years are given more weight.

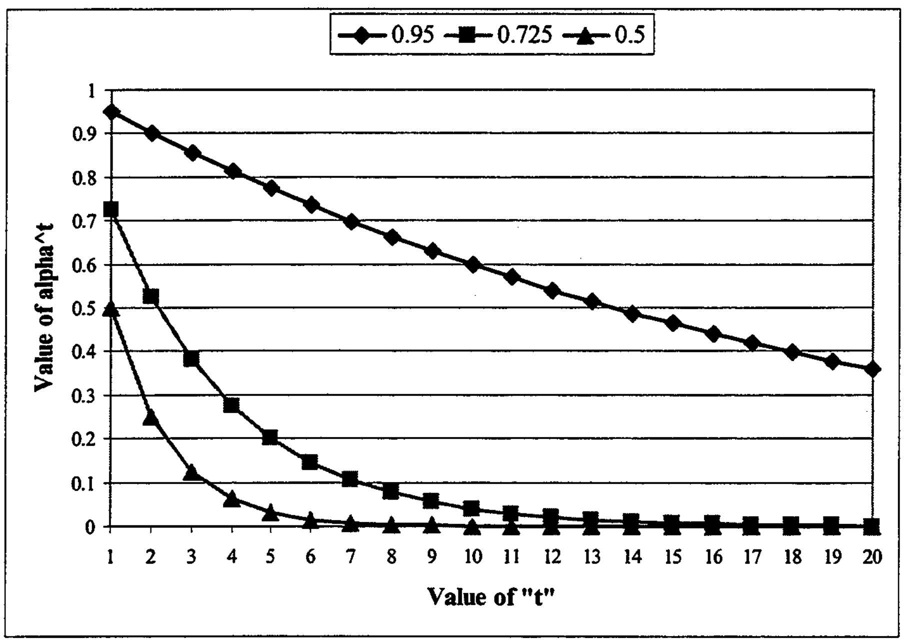

Figure 5.1 Different Values of α. Different values of α place a different weight on the annual deviations from targets.

Figures 5.2 and 5.3 summarize the effects of α and T on the optimal infinite horizon objective function values of IHMP using different approximation schemes. Figure 5.2 displays the changes in the optimal objective function values as T varies from 2010 to 2070 with $\alpha = 0.95$. Note that the optimal objective function values for truncation and dual equilibrium must be no larger than the true optimal objective function values of IHMP because discarding or aggregating constraints enlarges the feasible region on the approximating problems. Moreover, the objective function for the truncated problem does not include the squared deviation after year T, i.e., the component $\sum_{t>T} \sum_{(r,c)\in\Omega} \alpha^t f_t^{r,c}\left(X_t^{r,c}\right)$ is not part of the objective function. This makes truncation severely underestimate the true optimal objective function value for small T.

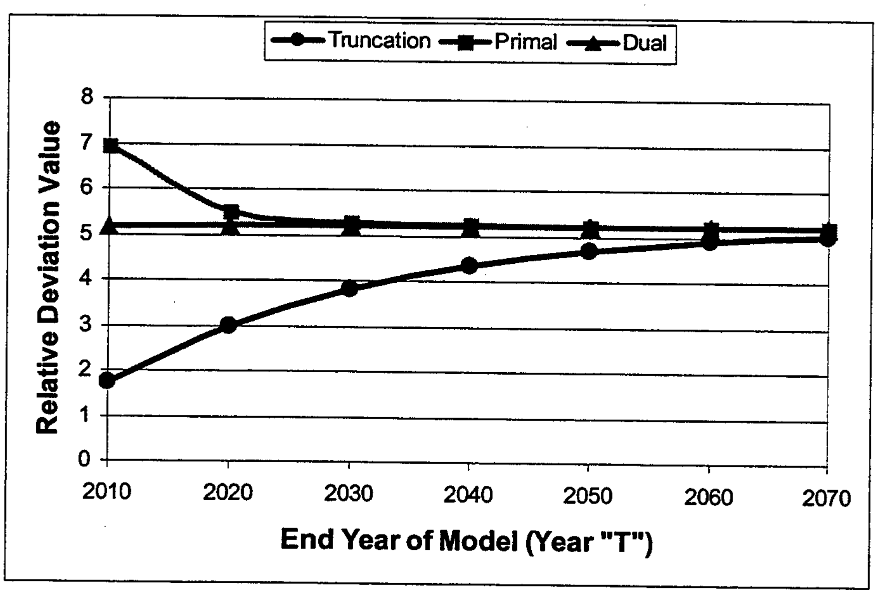

Figure 5.2 Convergence of Approximation Schemes, α = 0.95 (No PML). The above represents the effects of varying T on the objective function of truncation, primal, and dual equilibrium, for $\alpha = 0.95$. Truncation underestimates the optimal value for small values of T. Dual equilibrium appears robust for a wide range of values for the aggregation year T.

For primal equilibrium, forcing the decision variables after the equilibrium year to be the same as those in the equilibrium year produces a restriction of the infinite horizon problem. Therefore, the optimal objective function values for primal equilibrium cannot be smaller than the true optimal objective values. With all three approximation schemes, the optimal objective values of the approximating problems converge to the true optimal objective value for IHMP as T becomes large. However, dual equilibrium is the most robust in that it provides near optimal objective function values for all T values in Figure 5.2.

Figure 5.3 provides the same information as Figure 5.2 for $\alpha = 0.725$. Because this value of α puts essentially zero weight on target deviations in years 16 and beyond, all three approximation schemes converges to the true optimal solution at a faster rate when compared to Figure 5.2. The results for $\alpha = 0.5$ are similar and are not shown.

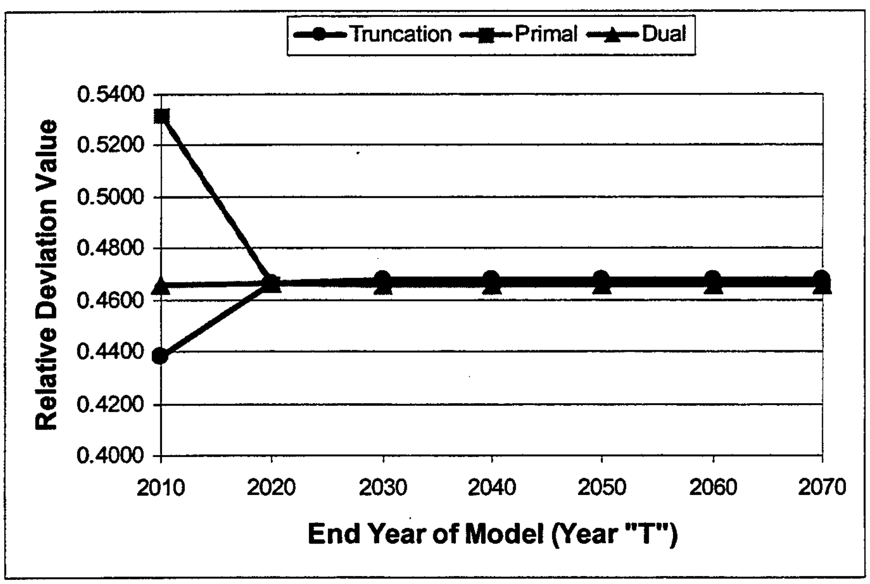

Figure 5.3 Convergence of Approximation Schemes, α = 0.725 (No PML). The above represents the effects of varying T on the objective function of truncation, primal, and dual equilibrium, for $\alpha = 0.725$. All three approximation schemes converge to the true optimal objective value faster than those in Figure 5.2.

When planning budgets and analyzing personnel policies or issues, analysts at DAPE-PRS focus on the next seven years. During FY 2000, the period between FY 2001 and 2007 is the basis for their analyses. To approximate the impact of end effects, the solution from primal equilibrium solution with $T = 2070$ is treated as the true optimal solution for IHMP.

Table 5.5 displays the relative errors in the inventory variables from the three approximation schemes for $T = 2010$ and $\alpha = 0.5$, 0.725, and 0.95. Again, dual equilibrium seems to be the most robust technique, for it yields the smallest amount of error. What is surprising here is the fact that the errors for primal equilibrium are worse than those for truncation when $\alpha = 0.725$ and 0.95. For $\alpha = 0.5$, weights on target deviations are essentially zero for FY 2008 and beyond. Thus, primal equilibrium and truncation are nearly equivalent and, as shown in Table 5.5, the errors for primal equilibrium and truncation are nearly the same.

	0.95		0.725		0.5	
	2001-2003	2004-2007	2001-2003	2004-2007	2001-2003	2004-2007
Truncation	0.17%	0.31%	0.01%	0.06%	0.02%	1.80%
Primal Eq	0.69%	2.53%	0.15%	1.97%	0.03%	1.76%
Dual Eq	0.05%	0.26%	0.00%	0.01%	0.00%	0.01%

Table 5.5 Relative Error for $T = 2010$ and $\alpha = 0.95$, 0.725, and 0.5. This table lists relative errors in the inventory variables for truncation, primal, and dual equilibrium with $T = 2010$ and varying values of α. When compared with the other two approximation schemes, dual equilibrium is the most robust. On the other hand, all three generate relatively small errors for the inventory variables during the first seven years of the planning horizon.

Similarly, Table 5.6 displays the relative errors for $\alpha = 0.725$ and $T = 2010$, 2020, and 2030. As before, dual equilibrium is the most robust technique, for it generates the least error. For the other schemes, the errors decrease as T increases. However, the primal equilibrium slightly outperforms truncation for larger T values, i.e., $T = 2020$ and 2030.

	2010		2020		2030	
	2001-2003	2004-2007	2001-2003	2004-2007	2001-2003	2004-2007
Truncation	0.01%	0.06%	0.00%	0.37%	0.00%	0.37%
Primal Eq	0.15%	1.97%	0.00%	0.00%	0.00%	0.00%
Dual Eq	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%

Table 5.6 Relative Error for $\alpha = 0.725$ and $T = 2010$, 2020, and 2030. This table lists relative errors in the inventory variables for $\alpha = 0.725$ and varying values of T. Dual equilibrium continues to be robust for varying values of T and primal equilibrium slightly outperforms truncation for $T = 2020$ and 2030. In general, the solution quality for all three schemes improves as T increases.

Primal equilibrium is used to access the solution quality of the sampling scheme. With current data, dual equilibrium is robust for a wide range of T values and it is not meaningful to combine the technique with sampling. Furthermore, the above results also suggest that truncation may be effective when α and T are small. However, a small T value does not lend itself to sampling.

Below, four sampling schemes are examined and they are

Scheme 1: Sample years 2001 to 2010, 2015, 2020, 2025, and 2030.

Scheme 2: Sample years 2001 to 2010, 2014, 2018, 2022, 2026, and 2030.

Scheme 3: Sample years 2001 to 2009, 2012, 2015, 2018, 2021, 2024, 2027, and 2030.

Scheme 4: Sample years 2001 to 2010, 2012, 2014, 2016, 2018, 2020, 2022, 2024, 2028, and 2030.

Table 5.7 displays the size of the approximating problem when primal equilibrium with $T = 2030$ is combined with the above sampling schemes.

	Sampling Scheme			
	1	2	3	4
Variables	907	970	1033	1285
Constraints	889	1011	1133	1621

Table 5.7 Size of Sampling Schemes. The above table displays the number of variables and constraints for the four sampling schemes.

When compared to the primal equilibrium solution, the errors due to the four sampling schemes are similar. (See Table 5.8) Errors in the inventory variables tend to be small and errors in variables representing promotion and accessions are larger. For promotions below the zone, the 13.7% error for schemes 1, 2, and 4, for FY 2002, are a result of errors for promotion to MAJ and COL (all career fields). Promotion to MAJ results in a difference of 14 officers, while for COL the difference for all career fields is at most 3 officers. This is not the case for accessions. The 17.7% error in FY 2006 is a difference of 798 officers. In any case, Table 5.8 suggests that sampling scheme 1 is adequate, for it generates similar errors as those requiring more frequent sampling and produces a smaller approximating problem.

	Error Results Scheme 1 (Sample size of 5)						
	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Inventory	0.3%	0.5%	0.6%	1.4%	2.0%	4.3%	**5.6%**
PZ	1.0%	1.0%	2.3%	4.6%	4.4%	4.3%	4.7%
BZ	0.6%	**13.7%**	**10.0%**	2.0%	3.7%	2.8%	**12.3%**
AZ	**6.2%**	1.5%	1.0%	**7.1%**	4.7%	4.9%	4.0%
ACCESS	0.0%	0.0%	0.0%	0.0%	0.0%	**17.7%**	**7.1%**
	Error Results Scheme 2 (Sample size of 4)						
	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Inventory	0.3%	0.5%	0.6%	1.4%	2.0%	4.3%	**5.6%**
PZ	1.0%	1.0%	2.3%	4.6%	4.7%	4.3%	4.7%
BZ	0.3%	**13.7%**	**10.0%**	2.0%	2.2%	**5.6%**	**12.4%**
AZ	**6.2%**	1.5%	1.0%	**7.1%**	**20.1%**	4.9%	4.0%
ACCESS	0.0%	0.0%	0.0%	0.0%	0.0%	**17.7%**	**7.1%**
	Error Results Scheme 3 (Sample size of 3)						
	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Inventory	1.0%	2.0%	2.8%	3.2%	3.6%	4.1%	3.1%
PZ	**5.3%**	5.0%	4.6%	4.6%	4.8%	4.4%	**10.6%**
BZ	2.4%	2.4%	4.9%	2.2%	3.7%	**7.1%**	**12.6%**
AZ	1.8%	4.7%	**26.0%**	**15.9%**	**20.1%**	**8.4%**	**9.1%**
ACCESS	0.0%	0.0%	0.0%	0.0%	0.0%	**8.0%**	**8.0%**
	Error Results Scheme 4 (Sample size of 2)						
	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Inventory	0.3%	0.5%	0.6%	1.4%	2.0%	4.3%	**5.6%**
PZ	0.9%	1.0%	2.3%	4.6%	4.7%	4.3%	4.7%
BZ	2.9%	**13.7%**	**10.0%**	2.0%	2.2%	**5.6%**	**12.4%**
AZ	**6.2%**	1.5%	1.0%	**7.1%**	**20.1%**	4.9%	**7.0%**
ACCESS	0.0%	0.0%	0.0%	0.0%	0.0%	**17.7%**	**7.1%**

Table 5.8 Error for Sampling Schemes (α = 0.95). When compared with primal equilibrium (T =2030 and α = 0.95) without sampling, all four sampling schemes generate errors in decision variables. Errors in the inventory variables are relatively small. Although relatively large, errors in above and below zone promotion typically correspond to the differences of 3 to 14 officers depending on their ranks and career fields.

C. APPLICATIONS

To illustrate the usefulness of IHMP as a decision aid, this section analyzes two personnel issues posed by DAPE-PRS analysts. One deals with the transformation of the Army's brigade structure and the other deals with the possible reduction in force hypothesized by the analysts. The first issue uses the primal equilibrium with sampling scheme 1, while the second issue relies on primal equilibrium with $T = 2030$. In both cases, the discount factor, α, is 0.95 in order to place more weight on years beyond FY 2007.

1. Transformation of Brigade Structure

In February 2000, the Secretary of the Army and the Army Chief of Staff announced their vision of a more strategically responsive, deployable, and lethal force. To support this vision, DAPE-PRS analysts pose a new force structure as displayed in Table 5.9. In particular, the ACC targets for LT and CPT increase annually by 23 and 40 officers, respectively. In addition, there is also an annual increase of 13 officers for the MAJ targets in the OP career field. Because the total number of ACC officers must remain the same as those in Table 5.1, DAPE-PRS analysts project that the Army will offset these increases by reducing the targets for MAJ-IS.

	2001	2002	2003	2004	2005	2006	$\geq$ 2007
LT	12853	12876	12899	12922	12945	12945	12945
CPT	16875	16915	16955	16995	17035	17035	17035
MAJ-OP	7694	7707	7720	7733	7746	7746	7746
MAJ-IS	641	565	489	413	337	337	337

Table 5.9 New Brigade Targets. This table lists targets for LT, CPT, MAJ-OP, and MAJ-IS, for the new brigade structure. Other targets are the same as those listed in Table 5.1.

The key issue is how many officers newly promoted to MAJ should be designated or assigned to the OP career field. The current plan is to designate 75% of the new MAJ to OP. Figure 5.4 displays the IHMP results under the current designation plan and the new targets. The inventory for MAJ-OP is below its target by approximately 5% in FY 2001 and it is approximately 17% below the target in FY 2007. For FY 2008 and beyond, the inventory for MAJ-OP can be as much as 25% below its targets.

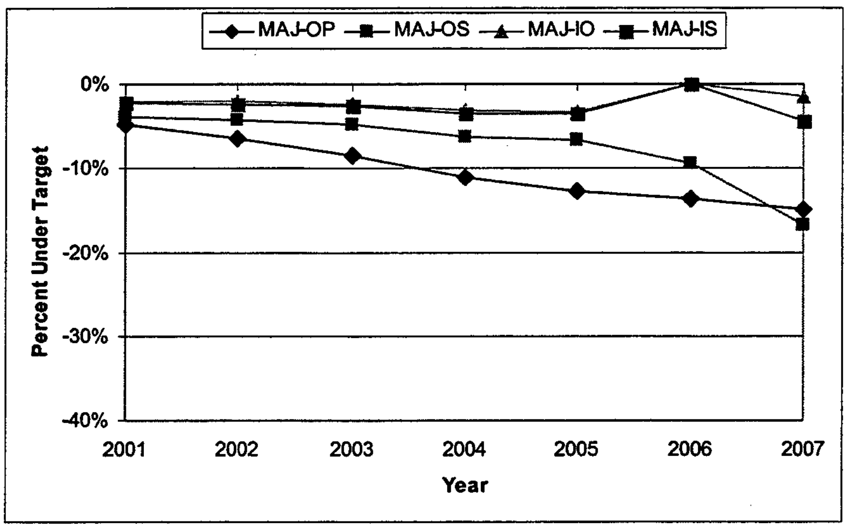

Figure 5.4 Impact of New Targets for MAJ-OP. As grade targets increase from 2001 to 2006, the inventory is increasingly under target. By year 2007, MAJ-OP is 15% under its target.

To properly test or 'field' the new structure, the shortages of MAJ-OP in Figure 5.4 is not acceptable. In an effort to determine the appropriate career field designation rates, a constraint is added to ensure that the MAJ-OP inventory does not fall below p percentage of its target. Figures 5.5 and 5.6 display the results from the modified IHMP with p equal to 90% and 95%, respectively. When requiring the MAJ-OP inventory to be no less than 90% of its targets (see Figure 5.5), the MAJ inventory for other career fields

is within 15% of their targets. When p is 95% (see Figure 5.6), other career fields miss their targets as much as 30% during the first seven years of the planning horizon.

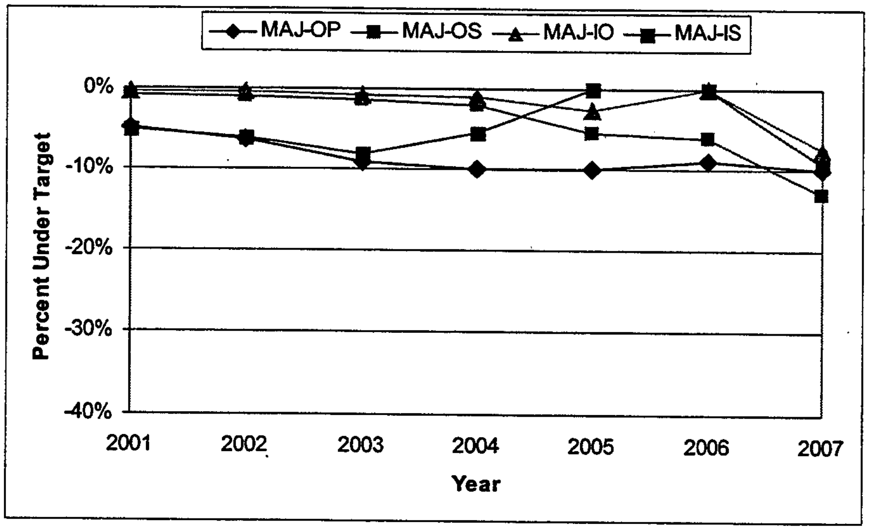

Figure 5.5 Requiring MAJ-OP Inventory to be at Least 90% of its Targets. The constraints on MAJ-OP inventory forces the MAJ inventory in other career fields to deviate from their targets by at most 15%.

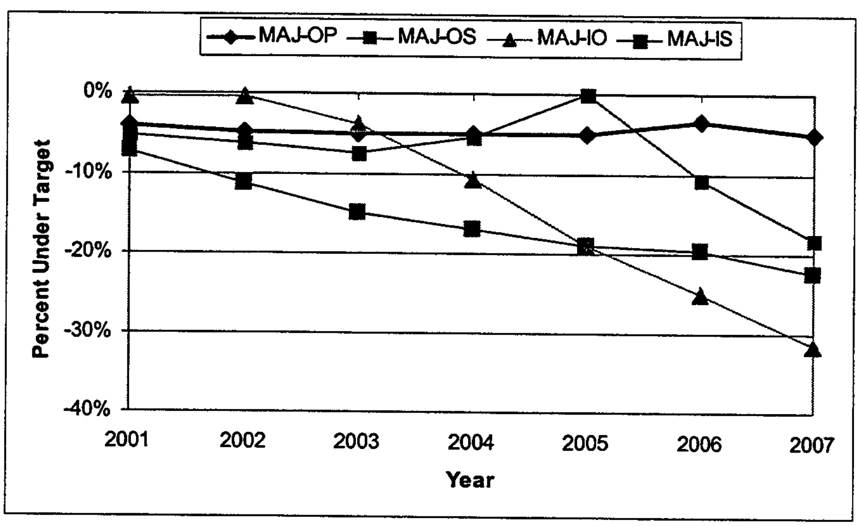

Figure 5.6 Requiring MAJ-OP Inventory to be at Least 95% of its Targets. The constraints on MAJ-OP inventory forces the MAJ inventory in other career fields to deviate from their targets quite drastically in FY 2007.

To summarize, the above two figures demonstrate that there are significant consequences associated with the change in force structure as envisioned by DAPE-PRS analysts. Assuming that the fielding of the new structure requires MAJ-OP inventory to be within 95% of its targets, outputs from IHMP also provide the appropriate designation rates for each career field. In Figure 5.7, IHMP suggests that the designation rate for MAJ-OP needs to be at approximately 79% in FY 2001 and increased to approximately 85% in FY 2007. These new rates represent significant increases in the current OP designation rate (75%) at the expense of a drastic reduction in the designation rates for other career fields.

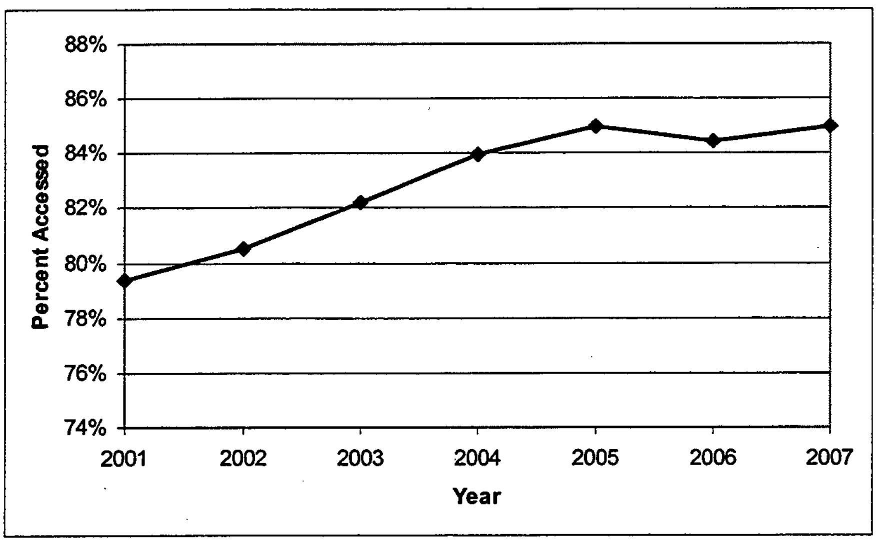

Figure 5.7 New MAJ-OP Designation Rates. MAJ-OP designation rates when the inventory must be above 95% of its targets. To achieve this inventory level, the Army must designate 79% to 85% of newly promoted MAJ to the OP career field.

2. Force Reduction

DAPE-PRS analysts postulate that the upcoming Quadrennial Defense Review may eliminate the possibility of a multiple regional conflict. In conjunction with the current shortages of officers and an emphasis on precision munition attacks, the Army

may have to reduce its force structure from 480,000 to 425,000. DAPE-PRS analysts expect this reduction to decrease the ACC inventory by 8,018 officers and re-adjust the inventory targets to those shown in Table 5.10.

	2001	2002	2003	2004	2005	2006	≥ 2007
LT	12830	12436	11778	11121	10721	10721	10721
CPT	16835	16317	15454	14592	14068	14068	14068
MAJ-OP	7681	7444	7051	6657	6418	6418	6418
MAJ-OS	1229	1191	1128	1065	1027	1027	1027
MAJ-IO	614	596	564	533	513	513	513
MAJ-IS	717	695	658	621	599	599	599
LTC-OP	4932	4780	4528	4275	4122	4122	4122
LTC-OS	789	765	724	684	659	659	659
LTC-IO	395	382	362	342	330	330	330
LTC-IS	460	446	423	399	385	385	385
COL-OP	1730	1676	1588	1499	1445	1445	1445
COL-OS	277	268	254	240	231	231	231
COL-IO	138	134	127	120	116	116	116
COL-IS	161	156	148	140	135	135	135

Table 5.10 Force Reduction Targets. This table shows updated targets to reduce the ACC officer inventory by 1,500 officers in FY 2002, 2,500 in FY 2003, 2,500 in FY 2004, and 1,518 in FY 2005.

There are two methods for achieving this force reduction, one is to access less and the other is to implement a separation or PML program. The reduction in the numbers of LT and CPT can be achieved in part by decreasing accession. However, this is not possible in the near term because accessions for FY 2001 to 2005 are fixed. On the other hand, PML program is the only method to reduce the number of field grade officers beyond normal attrition. Table 5.11 displays two PML schemes, 1% and 5%, for setting the maximum percentage of officers to be assigned to the separation program in each rank. Note that higher grades mean higher PML percentages in both schemes.

	Weight	1%	5%
LT	1	1%	5%
CPT	2	2%	10%
MAJ	2	2%	10%
LTC	3	3%	15%
COL	3	3%	15%

Table 5.11 Maximum Percentages for PML. The above table displays the maximum PML percentages allowed under the 1% and 5% schemes. Based on the weights in the second column, the maximum PML percentages increase with rank.

Figure 5.8 summarizes the results from IHMP with primal equilibrium and T = 2030. (Note that the sampling scheme is not used here because the Army implements separation programs only when necessary. Therefore, models with PML are not suitable for the sampling scheme. The graphs in this figure show that the two PML schemes are ineffective at reducing the force to the target level.

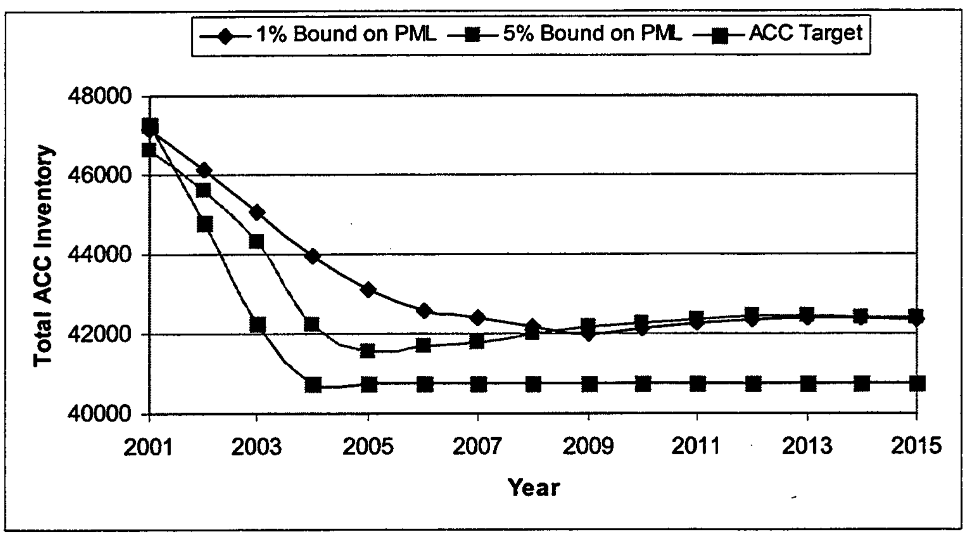

Figure 5.8 ACC Inventory Under a 1% and 5% PML. The two PML schemes cannot reduce the ACC officer inventory to the required targets.

For additional details on the deviations, Figure 5.9 shows that the number of CPT is more than 20% above its target in the long run. However, the field grade officer inventories are closer to their targets. In particular, the COL inventory matches its targets in FY 2010 and beyond.

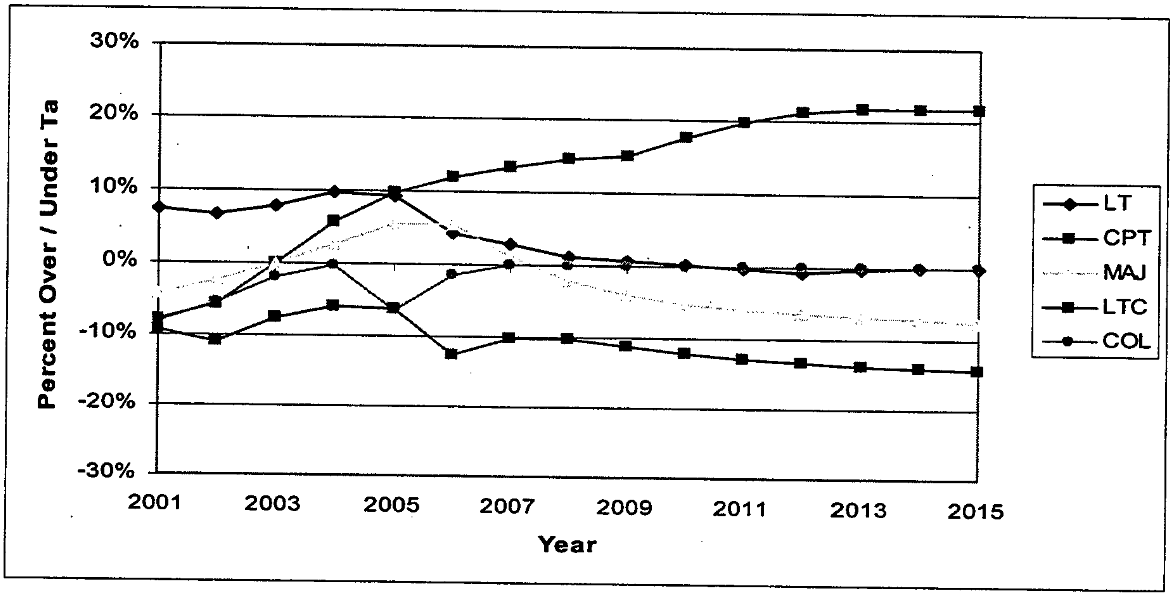

Figure 5.9 Grade Deviation Under the 5% PML Policy. The deviations for CPT increase from approximately –9% to around 22%. After FY 2009, the deviations for MAJ and LTC are around –8% and –14%, respectively. These graphs suggest that inventory targets may not be well balanced.

Upon further reflection, it is suspected that the targets may not be well aligned with the officer career path structure. Because the Army only accesses new officers as LT, there must be a sufficient number of LT and CPT in order to have the desired number of field grade officers. Figure 5.9 corroborates this conclusion, for it suggests that there need to be more CPT in order to support the desired number of field grade officers. To confirm this conclusion numerically, targets for CPT in FY 2007 and beyond are increased by approximately 20% and the ones for MAJ and COL are decreased by approximately 20%. Table 5.12 highlights the adjusted targets and Figure 5.10 displays

the ACC inventory using the two PML schemes and the new targets. The figure shows that both PML policies eventually reduce the ACC inventory to the desired level. However, the effect of the 1% PML policy is slightly more gradual.

	2001	2002	2003	2004	2005	2006	≥ 2007
LT	12830	12436	11778	11121	10721	10721	10721
CPT	16835	16317	15454	14592	14068	14068	16882
MAJ-OP	7681	7444	7051	6657	6418	6418	5134
MAJ-OS	1229	1191	1128	1065	1027	1027	822
MAJ-IO	614	596	564	533	513	513	410
MAJ-IS	717	695	658	621	599	599	479
LTC-OP	4932	4780	4528	4275	4122	4122	3298
LTC-OS	789	765	724	684	659	659	527
LTC-IO	395	382	362	342	330	330	264
LTC-IS	460	446	423	399	385	385	308
COL-OP	1730	1676	1588	1499	1445	1445	1445
COL-OS	277	268	254	240	231	231	231
COL-IO	138	134	127	120	116	116	116
COL-IS	161	156	148	140	135	135	135

Table 5.12 Adjusted Targets. For FY 2007 and beyond, targets for CPT were increased by roughly 20%, while targets for MAJ and LTC, were roughly decreased by 20%. These adjustments are shown in the shaded cells.

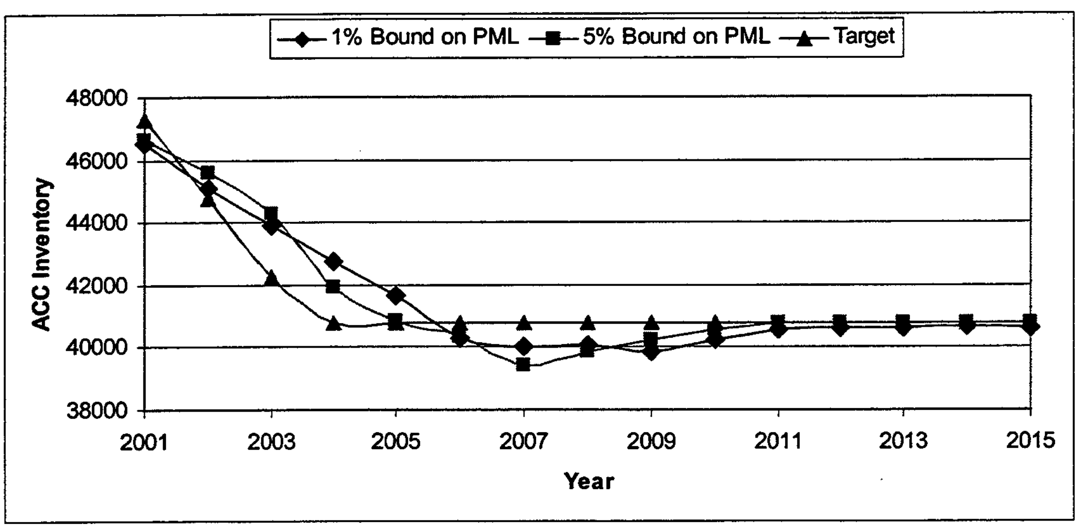

Figure 5.10 Effects of Redistribution of Targets. Using the adjusted targets for CPT, MAJ, and LTC, the two PML schemes drive officer inventory to the desired level. The 1% PML policy allows the inventory to reach 40,015 officers by FY 2007 more gradually.

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VI. CONCLUSIONS AND RECOMMENDATIONS

This thesis develops an optimization model called an Infinite Horizon Manpower Planning Model as a decision aid for forecasting and managing officer inventory. The model addresses the officer inventory at an aggregate level and utilizes a new technique as well as those that exist in the literature to reduce the model's size. These techniques include truncation, primal and dual equilibrium, and sampling.

Although the resulting problem is slightly more difficult to implement or modify, dual equilibrium is robust and produces near optimal solutions for a wide range of aggregation year values in our investigation. Comparing solution errors also reveals that primal equilibrium slightly outperforms truncation for larger truncation or equilibrium years. When combined with primal equilibrium, the sampling scheme is also effective at reducing the size of the primal equilibrium problem while maintaining essentially the same solution quality.

To illustrate the model's effectiveness as a decision aid, this thesis analyzes two personnel issues posed by DAPE-PRS analysts. One deals with the transformation of the Army's brigade structure and the other deals with a possible reduction in force. For the transformation of the brigade structure, the model is useful in quantifying the effects on other career fields when the number of MAJ in the OP career field is constrained to be above certain percentages of its annual inventory targets. Moreover, outputs from the model also suggest the appropriate designation rates for OP in order to meet the desired inventory requirements. For the potential reduction in force, results from the model suggest that the inventory targets provided by the analysts are not well aligned, because

targets for CPT are not sufficient to support targets for field grade officers. Instead of changing the career path structure, e.g., so that CPT are promoted to field grade officers in a shorter period, outputs from the model are used to better align the targets.

Currently, DAPE-PRS is planning to modify the Officer Aggregate system in order to improve its solution and address OPMS XXI. The optimization model in this system truncates the planning horizon at 20 years and accounts for officer inventory at a more detailed level. Based on the results herein, some combinations of the four approximation schemes may be applicable to this model and should be investigated.

APPENDIX A. PRIMAL EQUILIBRIUM FORMULATION

Below is a formulation of IHMP using the primal equilibrium approximation scheme. Many of the equations are the same IHMP equations from Chapter III with $\forall\, t$ replaced with $t \leq T$. Constraints and definitions for indices, data, and decision variables are either similar to those in Chapter III or as described in Chapter IV.

$$\text{Min} \sum_{(r,c)\in\Omega}\left[\sum_{t=1}^{T-1}\alpha^t f_t^{r,c}(X_t^{r,c}) + \frac{\alpha^T}{1-\alpha} f_T^{r,c}(X_T^{r,c})\right]$$

Subject to

Inventory constraints

$$X_t^{LT,NA} - (1 - lr_t^{LT,NA})\cdot X_{t-1}^{LT,NA} - A_t + PZ_t^{CPT,NA} + AZ_t^{CPT,NA} + PML_t^{LT,NA} = 0 \quad \forall\, 1 \leq t \leq T$$

$$X_T^{LT,NA} - (1 - lr_T^{LT,NA})\cdot X_T^{LT,NA} - A_T + PZ_T^{CPT,NA} + AZ_T^{CPT,NA} + PML_T^{LT,NA} = 0$$

$$X_t^{CPT,NA} - (1 - lr_t^{CPT,NA})\cdot X_{t-1}^{CPT,NA} - PZ_t^{CPT,NA} - AZ_t^{CPT,NA} + BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA} + PML_t^{CPT,NA} = 0 \quad \forall\, 1 \leq t \leq T$$

$$X_T^{CPT,NA} - (1 - lr_T^{CPT,NA})\cdot X_T^{CPT,NA} - PZ_T^{CPT,NA} - AZ_T^{CPT,NA} + BZ_T^{MAJ,NA} + PZ_T^{MAJ,NA} + AZ_T^{MAJ,NA} + PML_T^{CPT,NA} = 0$$

$$X_t^{MAJ,c} - (1 - lr_t^{MAJ,c})\cdot X_{t-1}^{MAJ,c} - CF_t^c + BZ_t^{LTC,c} + PZ_t^{LTC,c} + AZ_t^{LTC,c} + PML_t^{MAJ,c} = 0 \quad \forall\, 1 \leq t \leq T, c \neq NA$$

$$X_T^{MAJ,c} - (1 - lr_T^{MAJ,c})\cdot X_T^{MAJ,c} - CF_T^c + BZ_T^{LTC,c} + PZ_T^{LTC,c} + AZ_T^{LTC,c} + PML_T^{MAJ,c} = 0 \quad \forall\, c \neq NA$$

$$X_t^{LTC,c} - (1 - lr_t^{LTC,c})\cdot X_{t-1}^{LTC,c} - BZ_t^{LTC,c} - PZ_t^{LTC,c} - AZ_t^{LTC,c} + BZ_t^{COL,c} + PZ_t^{COL,c} + AZ_t^{COL,c} + PML_t^{LTC,c} = 0 \quad \forall\, 1 \leq t \leq T, c \neq NA$$

$$X_T^{LTC,c} - (1 - lr_T^{LTC,c}) \cdot X_T^{LTC,c} - BZ_T^{LTC,c} - PZ_T^{LTC,c} - AZ_T^{LTC,c}$$
$$+ BZ_T^{COL,c} + PZ_T^{COL,c} + AZ_T^{COL,c} + PML_T^{LTC,c} = 0 \qquad \forall\ c \neq NA$$

$$X_t^{COL,c} - (1 - lr_t^{COL,c}) \cdot X_{t-1}^{COL,c} - BZ_t^{COL,c} - PZ_t^{COL,c} - AZ_t^{COL,c}$$
$$+ bg_t + PML_t^{COL,c} = 0 \qquad \forall\ 1 \leq t \leq T,\ c \neq NA$$

$$X_T^{COL,c} - (1 - lr_T^{COL,c}) \cdot X_T^{COL,c} - BZ_T^{COL,c} - PZ_T^{COL,c} - AZ_T^{COL,c}$$
$$+ bg_T + PML_T^{COL,c} = 0 \qquad \forall\ c \neq NA$$

Promotion opportunity constraints

$$\underline{pr}_r \cdot (pze_t^{r,c} - BZ_{t-1}^{r,c}) \leq (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq \overline{pr}_r \cdot (pze_{t,yg}^{r,c} - BZ_{t-1}^{r,c})$$
$$\forall\ (r, c) \in \Omega,\ (r,c) \neq \{(LT,NA), (CPT,NA), (MAJ,OP), \ldots, (MAJ,IS)\},\ t \leq \tau_{r,c}$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{t-3} \leq (PZ_t^{CPT,NA} + AZ_t^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{t-3} \qquad \forall\ \tau_{CPT,NA} < t \leq T$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{T-2} \leq (PZ_T^{CPT,NA} + AZ_T^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{T-2}$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{T-1} \leq (PZ_T^{CPT,NA} + AZ_T^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{T-1}$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_T \leq (PZ_T^{CPT,NA} + AZ_T^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_T$$

$$\underline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_{t-1}^{CPT,NA} \leq (BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA}) \leq \overline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_{t-1}^{CPT,NA}$$
$$\forall\ \tau_{MAJ,NA} < t \leq T$$

$$\underline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_T^{CPT,NA} \leq (BZ_T^{MAJ,NA} + PZ_T^{MAJ,NA} + AZ_T^{MAJ,NA}) \leq \overline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_T^{CPT,NA}$$

$$\underline{pr}_r \cdot fyg_r \cdot X_{t-1}^{r-1,c} \leq (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq \overline{pr}_r \cdot fyg_r \cdot X_{t-1}^{r-1,c}$$
$$\forall\ r \in \{LTC, COL\},\ c \neq NA,\ \tau_{r,c} < t \leq T$$

$$\underline{pr}_r \cdot fyg_r \cdot X_T^{r-1,c} \leq (BZ_T^{r,c} + PZ_T^{r,c} + AZ_T^{r,c}) \leq \overline{pr}_r \cdot fyg_r \cdot X_T^{r-1,c}$$
$$\forall\ r \in \{LTC, COL\},\ c \neq NA$$

Below and Above zone promotion constraints

$$\underline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \le BZ_t^{r,c} \le \overline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \ne \{(LT,NA),\ (CPT,NA),(MAJ,OP),\ldots,(MAJ,IS)\},\ 2 \le t \le T$$

$$\underline{azp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \le AZ_t^{r,c} \le \overline{azp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \ne \{(LT,NA),(MAJ,OP),\ldots,(MAJ,IS)\},\ 2 \le t \le T$$

Career field accession constraints

$$\sum_{c \ne NA} CF_t^c - BZ_t^{MAJ,NA} - PZ_t^{MAJ,NA} - AZ_t^{MAJ,NA} = 0 \qquad \forall\ 2 \le t \le T$$

$$CF_t^c \ge cf_c \cdot (BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA}) \qquad \forall\ 2 \le t \le T, c \ne NA$$

Rolldown constraints

$$\sum_{c \ne NA} X_t^{COL,c} \le \sum_{c \ne NA} tgt_t^{COL,c} \qquad \forall\ 2 \le t \le T$$

$$\sum_{c \ne NA} X_t^{LTC,c} + \sum_{c \ne NA} X_t^{COL,c} \le \sum_{c \ne NA} tgt_t^{LTC,c} + \sum_{c \ne NA} tgt_t^{COL,c} \qquad \forall\ 2 \le t \le T$$

$$\sum_{c \ne NA} X_t^{MAJ,c} + \sum_{c \ne NA} X_t^{LTC,c} \le \sum_{c \ne NA} tgt_t^{MAJ,NA} + \sum_{c \ne NA} tgt_t^{LTC,c} \qquad \forall\ 2 \le t \le T$$

Program managed loss constraints

$$\sum_{\substack{(r,c) \in \Omega, \\ (r,c) \ne (MAJ,NA)}} PML_t^{r,c} \le \overline{pmlp}_r \cdot \sum_{\substack{(r,c) \in \Omega \\ (r,c) \ne (MAJ,NA)}} X_t^{r,c} \qquad \forall\ 2 \le t \le T$$

Accession constraints

$$\underline{acc}_t \le A_t \le \overline{acc}_t \qquad \forall\ 2 \le t \le T$$

Nonnegativity constraints

$$X_t^{r,c},\ PZ_t^{r,c},\ BZ_t^{r,c},\ AZ_t^{r,c},\ PML_t^{r,c},\ A_t,\ CF_t^c \ge 0 \qquad \forall\ r, c, 1 \le t \le T$$

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APPENDIX B. DUAL EQUILIBRIUM FORMULATION

Below is a formulation of IHMP using the dual equilibrium approximation scheme. Many of the equations are the same IHMP equations from Chapter III with $\forall\, t$ replaced with $t \le T$. Constraints and definitions for indices, data, and decision variables are either similar to those in Chapter III or as described in Chapter IV.

$$\text{Min} \sum_{(r,c)\in\Omega}\left[\sum_{t=1}^{T-1}\alpha^t f_t^{r,c}(X_t^{r,c}) + \frac{\alpha^T}{1-\alpha} f_T(\overline{X}_T^{r,c})\right]$$

Subject to

Inventory constraints

$$X_t^{LT,NA} - (1-lr_t^{LT,NA})\cdot X_{t-1}^{LT,NA} - A_t + PZ_t^{CPT,NA} + AZ_t^{CPT,NA} + PML_t^{LT,NA} = 0 \quad \forall\, 1 \le t < T$$

$$\begin{aligned}&\overline{X}_T^{LT,NA} - (1-lr_T^{LT,NA})\cdot[(1-\alpha)X_{T-1}^{LT,NA} + \alpha\overline{X}_T^{LT,NA}] - \\ &\quad \overline{A}_T + \overline{PZ}_T^{CPT,NA} + \overline{AZ}_T^{CPT,NA} + \overline{PML}_T^{LT,NA} = 0\end{aligned}$$

$$\begin{aligned}&X_t^{CPT,NA} - (1-lr_t^{CPT,NA})\cdot X_{t-1}^{CPT,NA} - PZ_t^{CPT,NA} - AZ_t^{CPT,NA} \\ &\quad + BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA} + PML_t^{CPT,NA} = 0 \qquad \forall\, 1 \le t < T\end{aligned}$$

$$\begin{aligned}&\overline{X}_T^{CPT,NA} - (1-lr_T^{CPT,NA})\cdot[(1-\alpha)X_{T-1}^{CPT,NA} + \alpha\overline{X}_T^{CPT,NA}] - \overline{PZ}_T^{CPT,NA} - \overline{AZ}_T^{CPT,NA} \\ &\quad + \overline{BZ}_T^{MAJ,NA} + \overline{PZ}_T^{MAJ,NA} + \overline{AZ}_T^{MAJ,NA} + \overline{PML}_T^{CPT,NA} = 0\end{aligned}$$

$$\begin{aligned}&X_t^{MAJ,c} - (1-lr_t^{MAJ,c})\cdot X_{t-1}^{MAJ,c} - CF_t^c \\ &\quad + BZ_t^{LTC,c} + PZ_t^{LTC,c} + AZ_t^{LTC,c} + PML_t^{MAJ,c} = 0 \qquad \forall\, 1 \le t < T, c \ne NA\end{aligned}$$

$$\begin{aligned}&\overline{X}_T^{MAJ,c} - (1-lr_T^{MAJ,c})\cdot[(1-\alpha)X_{T-1}^{MAJ,c} + \alpha\overline{X}_T^{MAJ,c}] - \overline{CF}_T^c \\ &\quad + \overline{BZ}_T^{LTC,c} + \overline{PZ}_T^{LTC,c} + \overline{AZ}_T^{LTC,c} + \overline{PML}_T^{MAJ,c} = 0 \qquad \forall\, c \ne NA\end{aligned}$$

$$\begin{aligned}&X_t^{LTC,c} - (1-lr_t^{LTC,c})\cdot X_{t-1}^{LTC,c} - BZ_t^{LTC,c} - PZ_t^{LTC,c} - AZ_t^{LTC,c} \\ &\quad + BZ_t^{COL,c} + PZ_t^{COL,c} + AZ_t^{COL,c} + PML_t^{LTC,c} = 0 \qquad \forall\, 1 \le t < T, c \ne NA\end{aligned}$$

$$\overline{X}_T^{LTC,c} - (1 - lr_T^{LTC,c}) \cdot [(1-\alpha) X_{T-1}^{LTC,c} + \alpha \overline{X}_T^{LTC,c}] - \overline{BZ}_T^{LTC,c} - \overline{PZ}_T^{LTC,c} - \overline{AZ}_T^{LTC,c} + \overline{BZ}_T^{COL,c} + \overline{PZ}_T^{COL,c} + \overline{AZ}_T^{COL,c} + \overline{PML}_T^{LTC,c} = 0 \qquad \forall\, c \neq NA$$

$$X_t^{COL,c} - (1 - lr_t^{COL,c}) \cdot X_{t-1}^{COL,c} - BZ_t^{COL,c} - PZ_t^{COL,c} - AZ_t^{COL,c} + bg_t + PML_t^{COL,c} = 0 \qquad \forall\, 1 \leq t < T, c \neq NA$$

$$\overline{X}_T^{COL,c} - (1 - lr_T^{COL,c}) \cdot [(1-\alpha) X_{T-1}^{COL,c} + \alpha \overline{X}_T^{COL,c}] - \overline{BZ}_T^{COL,c} - \overline{PZ}_T^{COL,c} - \overline{AZ}_T^{COL,c} + bg_T + \overline{PML}_T^{COL,c} = 0 \qquad \forall\, c \neq NA$$

Promotion opportunity constraints

$$\underline{pr}_r \cdot (pze_t^{r,c} - BZ_{t-1}^{r,c}) \leq (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq \overline{pr}_r \cdot (pze_{t,yg}^{r,c} - BZ_{t-1}^{r,c})$$
$$\forall\, (r, c) \in \Omega, (r,c) \neq \{(LT,NA), (CPT,NA), (MAJ,OP), \ldots, (MAJ,IS)\}, t \leq \tau_{r,c}$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot A_{t-3} \leq (PZ_t^{CPT,NA} + AZ_t^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot A_{t-3} \qquad \forall\, \tau_{CPT,NA} < t < T$$

$$\underline{pr}_{CPT} \cdot cr_3 \cdot [(1-\alpha)(A_{T-3} + \alpha A_{T-2} + \alpha^2 A_{T-1}) + \alpha^3 \overline{A}_T] \leq (\overline{PZ}_T^{CPT,NA} + \overline{AZ}_T^{CPT,NA}) \leq \overline{pr}_{CPT} \cdot cr_3 \cdot [(1-\alpha)(A_{T-3} + \alpha A_{T-2} + \alpha^2 A_{T-1}) + \alpha^3 \overline{A}_T]$$

$$\underline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_{t-1}^{CPT,NA} \leq (BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA}) \leq \overline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot X_{t-1}^{CPT,NA}$$
$$\forall\, \tau_{MAJ,NA} < t < T$$

$$\underline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot [(1-\alpha) X_{T-1}^{CPT,NA} + \alpha \overline{X}_T^{CPT,NA}] \leq (\overline{BZ}_T^{MAJ,NA} + \overline{PZ}_T^{MAJ,NA} + \overline{AZ}_T^{MAJ,NA}) \leq \overline{pr}_{MAJ} \cdot fyg_{MAJ} \cdot [(1-\alpha) X_{T-1}^{CPT,NA} + \alpha \overline{X}_T^{CPT,NA}]$$

$$\underline{pr}_r \cdot fyg_r \cdot X_{t-1}^{r-1,c} \leq (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq \overline{pr}_r \cdot fyg_r \cdot X_{t-1}^{r-1,c}$$
$$\forall\, r \in \{LTC, COL\}, c \neq NA, \tau_{r,c} < t < T$$

$$\underline{pr}_r \cdot fyg_r \cdot [(1-\alpha] X_{T-1}^{r-1,c} + \alpha \overline{X}_T^{r-1,c}] \leq (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c}) \leq \overline{pr}_r \cdot fyg_r \cdot [(1-\alpha) X_{T-1}^{r-1,c} + \alpha \overline{X}_T^{r-1,c}] \qquad \forall\, r \in \{LTC, COL\}, c \neq NA$$

Below and Above zone promotion constraints

$$\underline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq BZ_t^{r,c} \leq \overline{bzp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c})$$
$$\forall\, (r,c) \in \Omega, (r,c) \neq \{(LT,NA), (CPT,NA), (MAJ,OP), \ldots, (MAJ,IS)\}, 2 \leq t < T$$

$$\underline{bzp}_r \cdot (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c}) \leq \overline{BZ}_T^{r,c} \leq \overline{bzp}_r \cdot (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \neq \{(LT,NA),\ (CPT,NA),(MAJ,OP),\ldots,(MAJ,IS)\}$$

$$\underline{azp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c}) \leq AZ_t^{r,c} \leq \overline{azp}_r \cdot (BZ_t^{r,c} + PZ_t^{r,c} + AZ_t^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \neq \{(LT,NA),(MAJ,OP),\ldots,(MAJ,IS)\},\ 2 \leq t < T$$

$$\underline{azp}_r \cdot (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c}) \leq \overline{AZ}_T^{r,c} \leq \overline{azp}_r \cdot (\overline{BZ}_T^{r,c} + \overline{PZ}_T^{r,c} + \overline{AZ}_T^{r,c})$$
$$\forall\ (r,c) \in \Omega,\ (r,c) \neq \{(LT,NA),(MAJ,OP),\ldots,(MAJ,IS)\}$$

Career field accession constraints

$$\sum_{c \neq NA} CF_t^c - BZ_t^{MAJ,NA} - PZ_t^{MAJ,NA} - AZ_t^{MAJ,NA} = 0 \qquad \forall\ 2 \leq t < T$$

$$\sum_{c \neq NA} \overline{CF}_T^c - \overline{BZ}_T^{MAJ,NA} - \overline{PZ}_T^{MAJ,NA} - \overline{AZ}_T^{MAJ,NA} = 0$$

$$CF_t^c \geq cf_c \cdot (BZ_t^{MAJ,NA} + PZ_t^{MAJ,NA} + AZ_t^{MAJ,NA}) \qquad \forall\ 2 \leq t < T,\ c \neq NA$$

$$\overline{CF}_T^c \geq cf_c \cdot (\overline{BZ}_T^{MAJ,NA} + \overline{PZ}_T^{MAJ,NA} + \overline{AZ}_T^{MAJ,NA}) \qquad \forall\ c \neq NA$$

Rolldown constraints

$$\sum_{c \neq NA} X_t^{COL,c} \leq \sum_{c \neq NA} tgt_t^{COL,c} \qquad \forall\ 2 \leq t < T$$

$$\sum_{c \neq NA} \overline{X}_T^{COL,c} \leq \sum_{c \neq NA} tgt_T^{COL,c}$$

$$\sum_{c \neq NA} X_t^{LTC,c} + \sum_{c \neq NA} X_t^{COL,c} \leq \sum_{c \neq NA} tgt_t^{LTC,c} + \sum_{c \neq NA} tgt_t^{COL,c} \qquad \forall\ 2 \leq t < T$$

$$\sum_{c \neq NA} \overline{X}_T^{LTC,c} + \sum_{c \neq NA} \overline{X}_T^{COL,c} \leq \sum_{c \neq NA} tgt_T^{LTC,c} + \sum_{c \neq NA} tgt_T^{COL,c}$$

$$\sum_{c \neq NA} X_t^{MAJ,c} + \sum_{c \neq NA} X_t^{LTC,c} \leq \sum_{c \neq NA} tgt_t^{MAJ,NA} + \sum_{c \neq NA} tgt_t^{LTC,c} \qquad \forall\ 2 \leq t < T$$

$$\sum_{c \neq NA} \overline{X}_T^{MAJ,c} + \sum_{c \neq NA} \overline{X}_T^{LTC,c} \leq \sum_{c \neq NA} tgt_T^{MAJ,c} + \sum_{c \neq NA} tgt_T^{LTC,c}$$

Program managed loss constraints

$$\sum_{\substack{(r,c)\in\Omega,\\(r,c)\neq(MAJ,NA)}} PML_t^{r,c} \leq \overline{pmlp}_r \cdot \sum_{\substack{(r,c)\in\Omega\\(r,c)\neq(MAJ,NA)}} X_t^{r,c} \qquad \forall\, 2 \leq t < T$$

$$\sum_{\substack{(r,c)\in\Omega,\\(r,c)\neq(MAJ,NA)}} \overline{PML}_T^{r,c} \leq \overline{pmlp}_r \cdot \sum_{\substack{(r,c)\in\Omega\\(r,c)\neq(MAJ,NA)}} \overline{X}_T^{r,c}$$

Accession constraints

$$\underline{acc}_t \leq A_t \leq \overline{acc}_t \qquad \forall\, 2 \leq t < T$$

$$\underline{acc}_T \leq \overline{A}_T \leq \overline{acc}_T$$

Nonnegativity constraints

$$X_t^{r,c},\ PZ_t^{r,c},\ BZ_t^{r,c},\ AZ_t^{r,c},\ PML_t^{r,c},\ A_t,\ CF_t^c \geq 0 \qquad \forall\, r, c, 1 \leq t < T$$

$$\overline{X}_T^{r,c},\ \overline{PZ}_T^{r,c},\ \overline{BZ}_T^{r,c},\ \overline{AZ}_T^{r,c},\ \overline{PML}_T^{r,c},\ \overline{A}_T,\ \overline{CF}_T^c \geq 0 \qquad \forall\, r, c$$

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